

The Sun to planetary centre of mass distance is coherent with solar activity on the decade, centennial and millennium time scales when Planet 9 is included in the solar system.

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Abstract

Planet 9 is currently a hypothetical planet the orbital parameters of which are based on anomalous orbits of Kuiper Belt objects. The orbital parameters are such that, if Planet 9 exists, the theory of solar barycentric dynamics would be profoundly altered. We show that, with Planet 9 included in the solar system, barycentric theory is a much more effective predictor of solar activity on the decadal, centennial and millennium time scales. In particular the most elementary quantity of barycentric theory, the Sun to planet barycentre distance, is more coherent with decadal solar activity cycles and grand solar activity minima than barycentric distance without Planet 9 included. Further, barycentric theory including Planet 9 contains strong components at periods corresponding to the Hallstaatt and Gleissberg cycles whereas barycentric theory without Planet 9 shows no evidence of these cycles. A challenge that emerged during this study was the absence of the strongest component in barycentric theory, the Jose component at ~ 178 year period, from the spectra of millennium scale solar activity. This conundrum was solved by demonstrating that, during the transformation from solar motion to solar activity, the Jose component in solar motion was, in the spectrum of solar activity, split into multiple sidebands due to phase modulation by lower frequency cycles. The excellent fit to solar activity at multiple time scales by barycentric theory with Planet 9 included is itself supporting evidence for the existence of Planet 9, specifically by providing an estimate of the current heliographic longitudinal location and orbital period.

Keywords. Planet 9; solar barycentric dynamics; reconstructed sunspot number; grand solar minima

1. Introduction

One of the outstanding problems in solar physics is the quasi-periodic variation in solar activity, for example, the time variation of sunspot numbers. The approximately decadal variation, the Schwabe cycle of solar activity, was explained by Parker (1955) as the result of the solar magnetic dynamo cycling between poloidal and toroidal states with the maxima in sunspot numbers coinciding with the toroidal state. However, it is not well understood why the cycle

varies in length from about 8 to 14 years or why the number of sunspots varies so strongly from one sunspot maximum to the next or why solar activity can pass through intervals when, for several decadal cycles, the sunspot numbers at solar cycle maxima are very low, for example, during the Maunder Minimum, Charbonneau (2010). Further examples of solar variability are short term variations in the daily periodicity range e.g. ~ 154 day periodicity, Reiger et al (1984), in the annual range, e.g. 1.6 year quasi-biennial solar periodicity Charvatova (2007), in the centennial periodicity range, e.g. Abreu et al (2012), and millennial periodicity e.g. Steinhilber et al (2012).

There are two broad approaches to explaining solar variability. The conventional approach is to assume variability in solar activity arises entirely from internal processes in the Sun, primarily through stochastic forcing of the solar dynamo process, (Wang and Sheeley 2003, Usoskin et al 2009, Beer et al 2018), or through stochastic forcing of the modes of spherical harmonic oscillation in layers of the Sun such as the tachocline, (Zaqarashvili et al 2010, , Liang et al 2019). A problem with this approach is that, due to the requirement of stochastic forcing, the cycles of activity are predictable only in the sense of predicting an average periodicity over time, e.g. Wang and Sheeley (2003) and, as a result it is difficult to test the theories against future observations. The alternative approach is to assume an interaction between the planets and the internal variability of the Sun such that the internal variability is entrained to the planetary periodicity by the interaction. The cycle of planets coming into and out of alignment is predictable so this approach offers the prospect of predicting future solar activity. With eight planets, hypothetically interacting with solar processes, the periodicity is complex with multiple periodicities ranging from daily to the millennium range possible. Two types of Sun-planet interaction have been proposed. (1). The direct effect of the planets raising gravitational tides on the Sun e.g. (Hung 2007, Scafetta 2012, Stefani et al 2016), or the planets exerting variable torques on some non-spherical aspect of the Sun, e.g. (Abreu et al 2012, Wolff and Patrone 2010, Charbonneau 2013). The direct gravitational effect is, however, very small resulting in tides on the Sun of the order of only 1 mm. Whether tides of this magnitude could result in any significant acceleration on the Sun is regarded with some scepticism, Callebaut et al (2012). (2). The other type of planet – Sun interaction results from periodic motion of Sun about the solar system centre of mass or barycentre. It has been proposed that the cyclic movements of the Sun about the barycentre result in cyclic solar activity, e.g. (Jose 1965, Landscheidt 1999, Wolff and Patrone 2010), with short term cyclic solar activity associated with solar motion due to the more rapidly orbiting of the inner planets, e.g., Charvatova (2007), while decadal solar activity, e.g., Jose (1965), is associated with the slowly moving outer planets.

Jose (1965) calculated, for the years 1653 to 2060, the distance between the Sun and the solar system barycentre and various other quantities such as the angular momentum of the Sun about the barycentre. Jose was able to show a reasonable correlation between the variation of

these macroscopic quantities and the record of sunspot activity available at that time, up to 1964. However, comparison of the timing of solar motion with the occurrence of more recent decadal solar cycles proved unconvincing, Fairbridge and Shirley (1987). As a result subsequent studies of the connection between the Sun's motion about the barycentre and solar activity shifted to comparisons of barycentric motion with solar activity on the centennial scale, specifically the occurrence of grand solar minima and maxima, e.g. Fairbridge and Shirley (1987). The problem with predicting solar activity on the centennial to millennial scale is that while the solar- barycentre distance varies strongly over the decadal time scale it is essentially constant, on average, over longer time scales such as centennial and millennial scales, see Fairbridge and Shirley (1987), Figure 3, or Cionco and Pavlov (2018), Figure 3. Thus, in attempting to predict the occurrence of grand solar minima, or even longer cycles of solar activity, for example, the approximately 1000 year Eddy cycle and the approximately 2400 year Hallstaatt cycle, from solar barycentric motion it has been necessary to resort to microscopic quantities associated with solar motion. For example, finding differences in the patterns of barycentric motion, (Fairbridge and Shirley 1987, Shirley 2009) or differences in the states of motion, for example states of ordered or chaotic motion, Charvatova (2009). As a more recent example, in an attempt to discover the long term, approximately 2400 year period, Hallstaatt cycle in solar barycentric motion, Scafetta et al (2016) found that a very small variation in the long term average eccentricity of the orbit of the planetary mass centre relative to the Sun has an approximately 2300 year period that could be associated with the Hallstaatt cycle in solar activity. Cionco and Soon (2015) used planetary barycentre dynamics in an attempt to relate the occurrence of extremely sharp decreases in stored potential energy in the Sun, that occasionally occur within the time intervals of grand solar minima, with producing the grand minima events. The problem with this approach is that the sharp decreases in potential energy that occur during the very close approaches of the Sun to the barycentre persist over intervals of only a few months whereas grand solar minima persist over intervals of several decades. Further, examination of the changes in potential energy as calculated by Cionco and Soon (2015) show that the sharp decreases in stored potential energy are invariably closely accompanied by somewhat broader increases in stored potential energy so that when averaged over intervals commensurate with the time span of grand solar minima the stored potential energy remains constant, i.e., there is no average decrease in stored potential energy commensurate with the average decreases in sunspot number that characterise grand solar minima.

Thus, the situation is, on one hand, the variation of solar activity by stochastic forcing of solar internal modes of oscillation negates the possibility of predicting solar activity other than in an "average periodicity over time" sense, making it difficult to test the theories against future predictions. On the other hand, the generation of cyclic solar activity from solar barycentric motion appears dubious for short term prediction and, over the long term, average solar

barycentric motion does not vary enough, other than in the sense of different patterns of the motion, to be useful for longer term prediction.

In this paper we consider the possibility that prediction of solar activity based on solar motion induced by the planets has proven inadequate because current solar barycentric dynamics does not include the effect of a, currently hypothetical, distant planet, the so called Planet 9, Brown and Batygin (2021). The existence of Planet 9 was proposed to explain the apparently anomalous orbits of several Kuiper Belt objects. However, Planet 9 has not been observed directly and there is ongoing controversy in the astronomical community as to its existence, Shankman et al (2017). However, if Planet 9 does exist, the mass and orbital radius indicated by Planet 9's effect on Kuiper Belt objects is such that Planet 9 would have a profound effect on solar barycentric dynamics and on the prediction of effects of planetary motion on solar activity. Therefore, in view of the past and continuing research in barycentric dynamics and the evident difficulty of applying the current eight planet formulation of barycentric dynamics to predicting solar activity it is timely to consider if the inclusion of a ninth planet in the solar system would improve the predictive capability of barycentric dynamics for solar activity. A corollary is, that if a nine planet formulation of barycentric dynamics proves more adequately predictive of solar activity than the current eight planet formulation, this in itself would provide additional indirect evidence for the existence of Planet 9.

Section 2 of the paper briefly discusses the current knowledge of Planet 9. Section 3 outlines the data sources and the simplified method of calculating the time variation of the barycentric quantities used in the paper. Section 4 demonstrates that solar barycentric motion with Planet 9 included provides a much better fit to the recent decadal variation in solar activity than barycentric motion without Planet 9. Section 5 uses the observed sunspot activity from 1610 to the present to show that recent grand solar minima are associated with macroscopic changes in Sun-planet barycentre distance. Section 6 uses the reconstructed sunspot number data, -6755 to 1885, to demonstrate that, in the millennium scale variation of planetary barycentric motion with Planet 9, there are significant components at the Hallstaatt and Gleissberg cycle periodicities that are absent in planetary barycentric motion without Planet 9. Additionally, it is shown that the orbital period of Planet 9 can be tuned to fit the Hallstaatt component of barycentric motion to previously observed estimates of the Hallstaatt cycle period. Section 7 addresses the conundrum that the dominant periodicity in planet barycentre motion, the Jose periodicity at about 177 year period, is, apparently, absent from the observed periodicity of millennium scale solar activity. The conundrum is resolved by demonstrating that phase modulation by longer period cycles splits the Jose component into multiple sidebands that contain all the power of the Jose component resulting in the absence, in solar activity spectra, of the Jose component itself. Further, it is shown that the phase of both the Hallstaatt component and the Jose component can be readily determined from the spectral content of

the reconstructed sunspot number. Section 8 briefly discusses how the observation of improved prediction of solar activity at multiple time scales by barycentric motion with Planet 9 included provides additional indirect evidence of the location and orbital period of Planet 9. Section 9 is a discussion that touches on the relevance of the Jose cycle in the occurrence of grand minima in solar activity and a conclusion that the discovery of the Jose periodicity in millennium scale solar activity may result in a greater awareness of the relevance of planet motion to cyclic solar activity.

2. Prior evidence of Planet 9.

Evidence for the existence of Planet 9 has been accumulating for about 20 years, (Bailey et al 2016, Batygin et al 2019). Analysis of the evidence, the unexplained clustering of Kuiper belt objects, suggests the existence of a new planet of mass 5 – 10 Earth masses, in an orbit of eccentricity 0.2 – 0.5 with a semi major axis of 400 - 800 AU inclined at 15 to 25 degrees to the ecliptic plane. Recently, the orbital parameters of this hypothetical planet have been narrowed by further analysis, Brown and Batygin (2021). However, the parameter estimates, based just on the anomalous clustering of distant objects, remain very uncertain.

If Planet 9 exists, the mass and distance from the Sun will radically alter the barycentric dynamics of the solar system, in particular, (1), the distance between the Sun and the solar system centre of mass and, (2), the angular momentum of the Sun about the solar system centre of mass. As these quantities were basic to previous attempts to demonstrate planetary influence on solar activity it is clear that previous work and predictions in this area should be revisited taking into account the possible existence of Planet 9.

3. Methods and data sources

3.1 Data sources.

Planet heliographic longitudes were obtained from <https://omniweb.gsfc.nasa.gov/coho/helios/heli.html>. Group sunspot numbers 1610 to 2015, Svalgaard and Schatten (2016), were obtained from <https://svalgaard.leif.org/research/gn-data.htm>. The reconstructed sunspot number between -6755 and 1885, Wu et al (2018) was obtained from https://www2.mps.mpg.de/projects/sun-climate/data/SN_composite.txt

3.2 Method of calculating solar barycentric dynamics including Planet 9.

The orbital parameters of Planet 9 are, at present, best estimates based on the anomalous orbital distribution of eccentric Kuiper belt objects. In view of the limited accuracy relating to the orbital parameters of Planet 9, in this article, the centre of mass of the planetary system is

calculated from a simplified model of planetary motion where all the planets move about the Sun in circular orbits in the ecliptic plane. This is a reasonable approximation for the known planets other than Mercury, the orbit of which is quite elliptical. As shown below, the barycentric dynamics of the eight known planets, calculated using circular orbits, is scarcely distinguishable from the barycentric dynamics calculated by more complex methods involving planet orbits based ephemeris data. The parameters of the planets used in the calculation are given in Table 1.

Table 1. Parameters of the planets. (Planet 9 table 1 planet parameters.docx)

Planet	Mass in mass Earth units, m_i	Radius, AU, r_i	Period, days, T_i	HGILongitude 01/01/1965, L_i	ELongitude 01/01/1965
Jupiter	318	5.2	4332	340.2	56.1
Saturn	95	9.5	10759	260.6	336.5
Venus	0.815	0.72	224.7	146.2	222.1
Earth	1	1	365.256	24.9	100.8
Neptune	17.15	30.07	60195	152.6	228.5
Uranus	14.53	19.19	30688	86.7	162.6
Planet 9	7	380	2706193	50 (fitted)	125.9 (fitted)

Note that for Planet 9 the mass, $m_9 = 7m_E$, radius, $r_9 = 380$ AU, and period, $T_9 = 2706193$ days, 7409 years, are taken from the best estimates of Brown and Batygin (2021), while the heliographic longitude in the ecliptic plane is used as an adjustable parameter to fit the Sun to planetary centre of mass distance, r_{CM9} , to observed solar activity, in this paper indicated by the variation of F10.7 cm radio flux, from 1965 to 2020, during solar cycles 20 to 24, Section 4.

The time variation of the coordinates, (x_i, y_i) , of the planets relative to the Sun as origin at (0,0) are calculated using

$$x_i = r_i \cos(\omega_i t + \phi_i), \quad y_i = r_i \sin(\omega_i t + \phi_i), \quad (1)$$

where the angular frequency $\omega_i = 2\pi/T_i$, the phase angle in radians, $\phi_i = (\pi/180)L_i$, and L_i is the heliographic longitude in degrees of the i th planet on January 01, 1965.

The coordinates, (x_{CM}, y_{CM}) , of the planetary centre of mass are given by

$$x_{CM} = \Sigma(m_i x_i) / \Sigma(m_i), \quad y_{CM} = \Sigma(m_i y_i) / \Sigma(m_i) \quad (2)$$

and the distance between the Sun and the planetary centre of mass, r_{CM} , is given by

$$r_{CM} = (x_{CM}^2 + y_{CM}^2)^{1/2} \quad (3)$$

The distance between the Sun and the solar system centre of mass, R_{CM} , is

$$R_{CM} = r_{CM}[\Sigma(m_i)/\{\Sigma(m_i) + m_{SUN}\}] = 0.00135r_{CM} \quad (4)$$

The velocity of the i th planet is calculated using

$$v_{x_i} = -\omega_i r_i \sin(\omega_i t + \phi_i), \quad v_{y_i} = \omega_i r_i \cos(\omega_i t + \phi_i)$$

The velocity of the planetary centre of mass is found from

$$v_{x_{CM}} = \Sigma(m_i v_{x_i})/\Sigma(m_i), \quad v_{y_{CM}} = \Sigma(m_i v_{y_i})/\Sigma(m_i)$$

The angular momentum, L_{CM} , of the planets centre of mass about the Sun is given by

$$L_{CM} = x_{CM} v_{y_{CM}} - y_{CM} v_{x_{CM}} \quad (5)$$

The angular momentum does not include the z components used in more precise estimates of angular momentum as, in the model used here, all planets orbit in the ecliptic plane, i.e. the orbital inclination and the z coordinate of each planet is zero.

The rate of change of planetary angular momentum, the torque, T_{CM} , about the Sun is given by

$$T_{CM} = dL_{CM}/dt \quad (6)$$

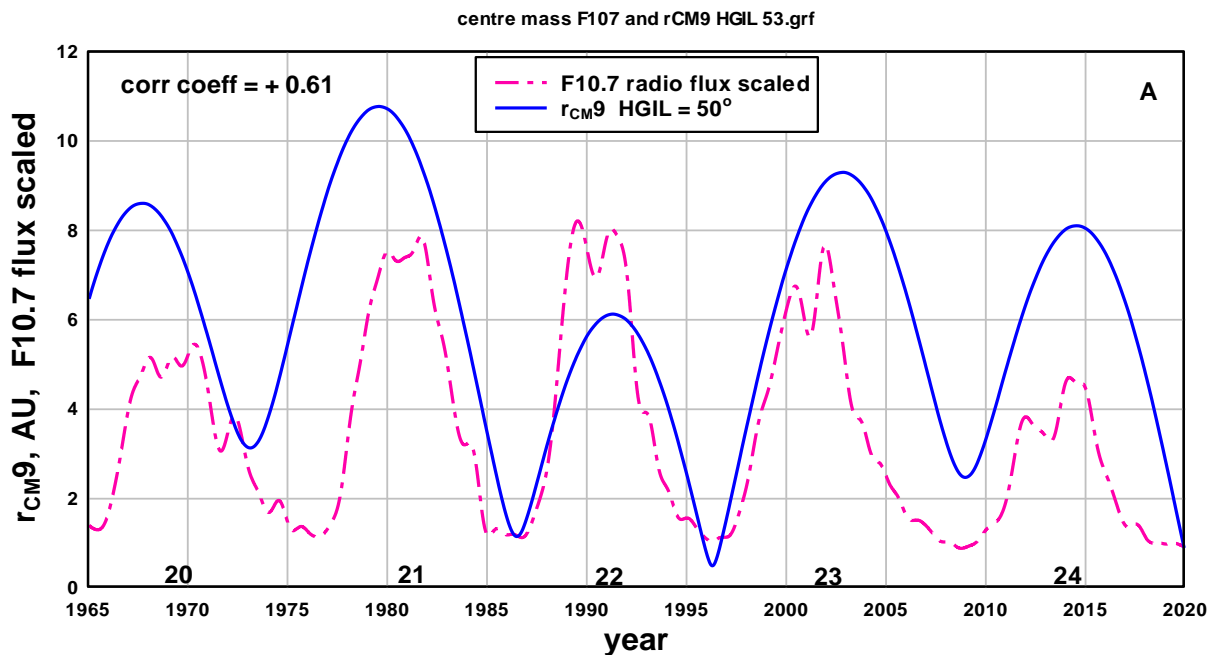
Usually, the calculation of barycentric dynamics is based on planet coordinates obtained from one of the several ephemeris available e.g. the JPL HORIZONS ephemeris. However, as the parameters of one of the planets, Planet 9, to be included in the calculation have very limited accuracy, Brown and Batygin (2021), and ephemeris data does not exist for Planet 9, the simple calculation as outlined above is regarded as adequate. We show in the next Section that the calculation of r_{CM} , due to the known eight planet system with this simple circular orbit method provides a time variation of r_{CM} that is difficult to distinguish by visual examination from the time variation of r_{CM} obtained using ephemeris data, (Jose 1965, Fairbridge and Shirley 1987, Javaraiah 2005)). We use, mainly, the simplest quantity, the time variation of the Sun to planet barycentre distance r_{CM} , equation 3, in this paper. However, r_{CM} and the angular momentum, L_{CM} , have very similar time dependence, Javaraiah (2005).

4. Decadal solar activity and the relation to solar barycentric motion including Planet 9.

4.1 Sun to planetary barycentre distance with Planet 9 included

The concept that solar barycentric motion is related to solar activity has gained very little acceptance within the solar physics community since it was first proposed by Jose (1965). The hypothesis of this paper is that barycentric dynamics that does not include Planet 9 is simply incorrect. A further hypothesis is that solar activity arises, either directly, Wolff and Patrone

(2010), or indirectly, due to the effect of short and medium term torques on the Sun due to the motions of the inner planets. We shall show that the short term planetary torques are greatest when the planetary angular momentum, L_{CM} , is at a maximum. Equation 5 indicates that L_{CM} is the product of barycentric distance and speed so L_{CM} is a maximum when r_{CM} is a maximum. Therefore, in the first instance, it is expected that solar activity as represented by, for example, sunspot number or F10.7 cm radio flux, should be directly proportional r_{CM} . In this section we use that expected relationship to fit the adjustable parameter of the Planet 9 orbit, the Planet 9 longitude, to the solar activity, as measured by F10.7 cm radio flux, during the most recent solar cycles 20 to 24. The time interval from 1965 to present is used as January 01, 1965 is the earliest time for planet longitudes available on the Omniweb site. The approach here is to fit the r_{CM9} variation to the solar activity variation by adjusting the heliographic longitude, HGIL9, of Planet 9. In Figure 1A we see that a Planet 9 longitude of 50 degrees provides a maximum correlation coefficient between the r_{CM9} variation and solar activity of + 0.61. The approach to this maximum correlation as longitude is varied is broad with the correlation falling to 0.55 at about +/- 10 degrees of 50 degrees. So the result is expressed as HGIL9 = 50 +/- 10 degrees.



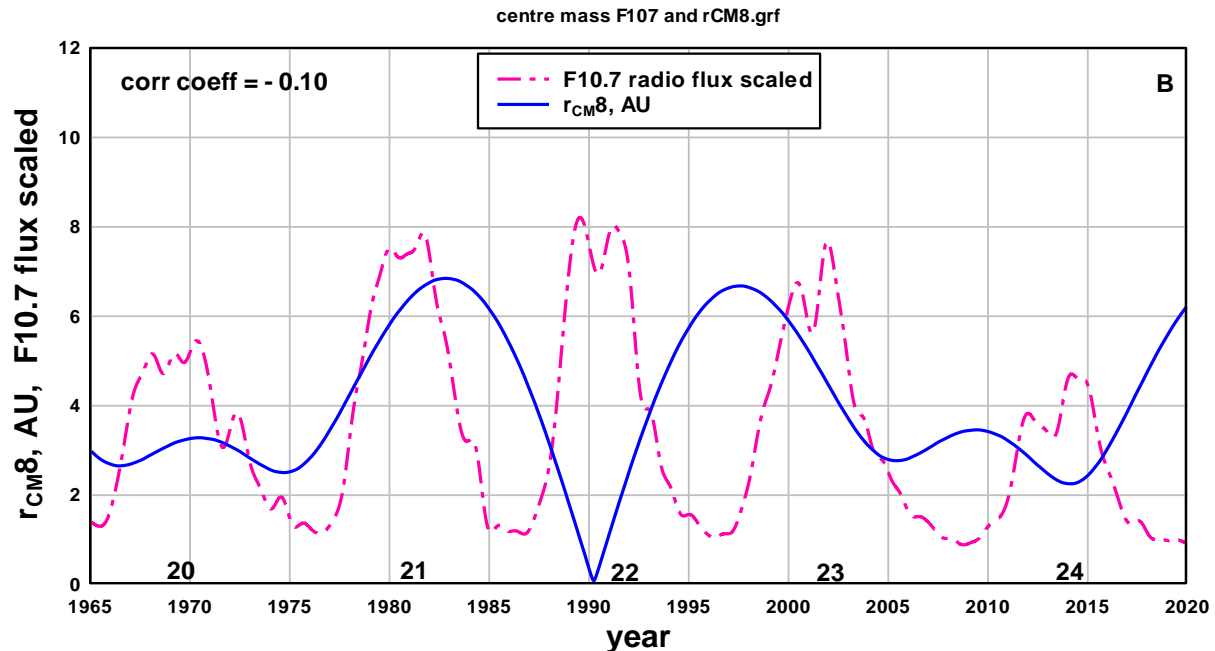


Figure 1. (A) Compares the time variation of Sun to planetary centre of mass distance, r_{CM9} , with the variation of F10.7 cm radio flux during solar cycles 20 – 24. (B) Compares the time variation of Sun to planetary centre of mass distance, r_{CM8} , with the variation of F10.7 cm radio flux during solar cycles 20 – 24. The coherence of r_{CM9} with the F10.7 variation is evident whereas the r_{CM8} variation is clearly non-coherent with F10.7 radio flux.

We next compare the solar activity with the planetary centre of mass distance, r_{CM8} , calculated with the known planets, Figure 1B. Here, as the orbital parameters of the eight planets are known exactly, there are no adjustable orbital parameters. From Figure 1B, the correlation coefficient between r_{CM8} and solar activity is -0.1 . This means, essentially, for decadal solar activity during the five solar cycles, the expected solar activity as calculated from planetary dynamics with the known planets is a very poor fit to the observed solar activity variation.

The excellent fit between r_{CM9} and solar activity and the very poor fit between r_{CM8} and solar activity supports the hypothesis that Planet 9 is essential for the assessing the relationship between planetary dynamics and solar activity.

As mentioned earlier in this section a secondary hypothesis is that solar activity arises mainly from the short term effects of the motion of the inner planets on the Sun. Here we reproduce Figure 1A with the inclusion of the rate of change of angular momentum, i.e. the planetary torque $T = dL/dt$ about the Sun, calculated using equations 5 and 6 for the nine planet system, Figure 2. An obvious feature is that the amplitude of the short term variations in torque, T , correlate closely in time with the r_{CM9} maxima and with the solar activity maxima.

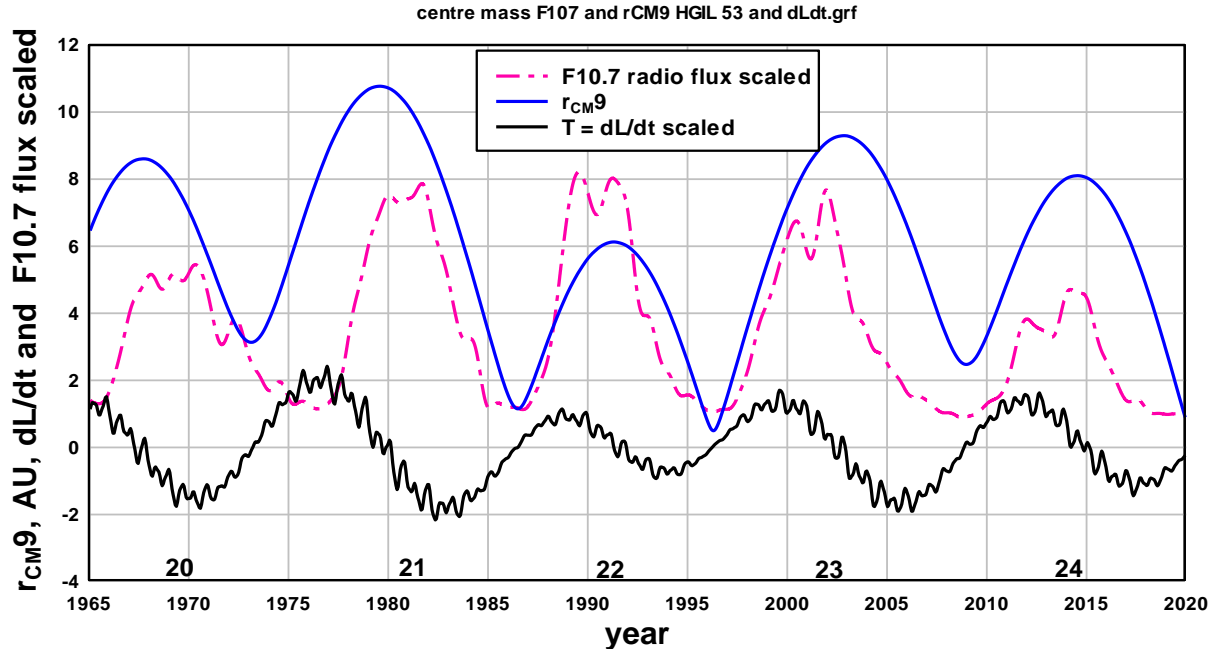


Figure 2. Compares the variation of r_{CM9} and F10.7 radio flux with the rate of change of angular momentum or torque on the planetary system centre of mass. The short term variations in torque due to the motion of the inner planets increase in amplitude as the Sun to centre of mass distance r_{CM9} increases and the short term variations coincide closely with the F10.7 radio flux measure of solar activity.

4.2 Planetary barycentre orbital motion with Planet 9 included.

Centre of mass orbital diagrams plot the time development of the planetary centre of mass location, (x_{CM}, y_{CM}) . Figure 3 shows the time dependence of planetary centre of mass location, between 1965 and 2035 for the eight planet system and the nine planet system. The inclusion of Planet 9 displaces the planetary centre of mass from the Sun. From Figures 1 and 2, we assess the average r_{CM9} distance is 6.73 AU and the average r_{CM8} distance is 4.13 AU. Using equation 4, the average distance of the solar system centre of mass from the Sun is $4.13 \times 0.00135 = 0.00557$ AU for the eight planet system and $6.73 \times 0.00135 = 0.00908$ AU for the nine planet system. Other than this additional displacement of the planetary centre of mass from the Sun, Figure 3 shows that during the interval 1965 to 2035, the pattern of centre of mass movement in space for the nine planet system is similar to the pattern of movement for the eight planet system. That is, the pattern of planetary centre of mass motion, primarily due to Jupiter, Saturn, Neptune and Uranus, is displaced outwards from the Sun position, $(0,0)$, with the inclusion of Planet 9. However, as shown in the previous section it is the displacement of the planetary centre of mass that profoundly affects the correlation between solar activity and barycentric dynamic quantities.

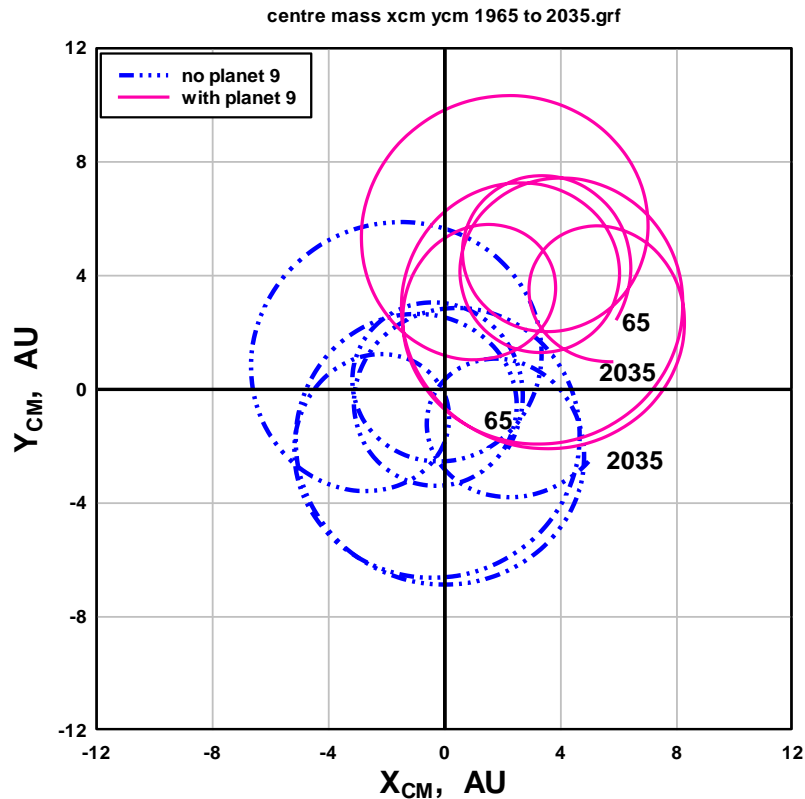


Figure 3. The orbital pattern for the planetary centre of mass relative to the Sun position at (0,0). The inclusion of Planet 9 in the planet system displaces the planetary centre of mass from the Sun by, a long term average of, 2.6 AU.

The orbital pattern for the planetary system with Planet 9 rotates about the Sun position, (0,0), at an angular rate corresponding to the period of Planet 9. In this paper, following Brown and Batygin (2021), T_9 is taken as 7,409 years, and the angular rate of rotation of the pattern in Figure 3 about the Sun at (0,0) is 5 degrees/century.

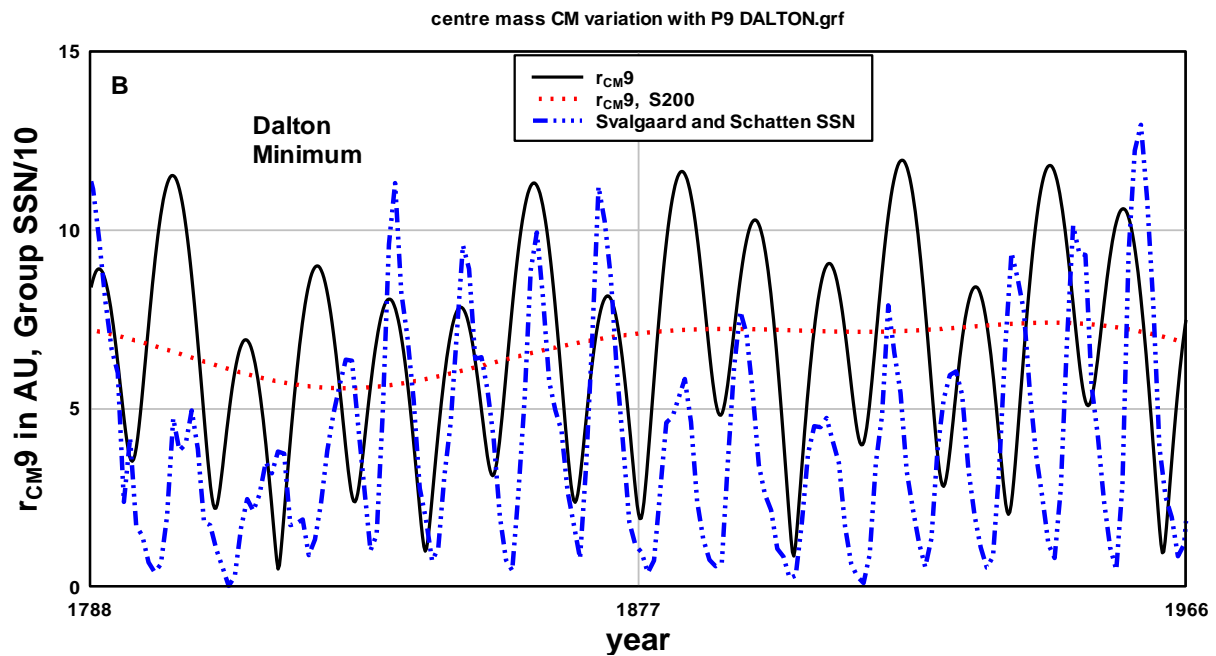
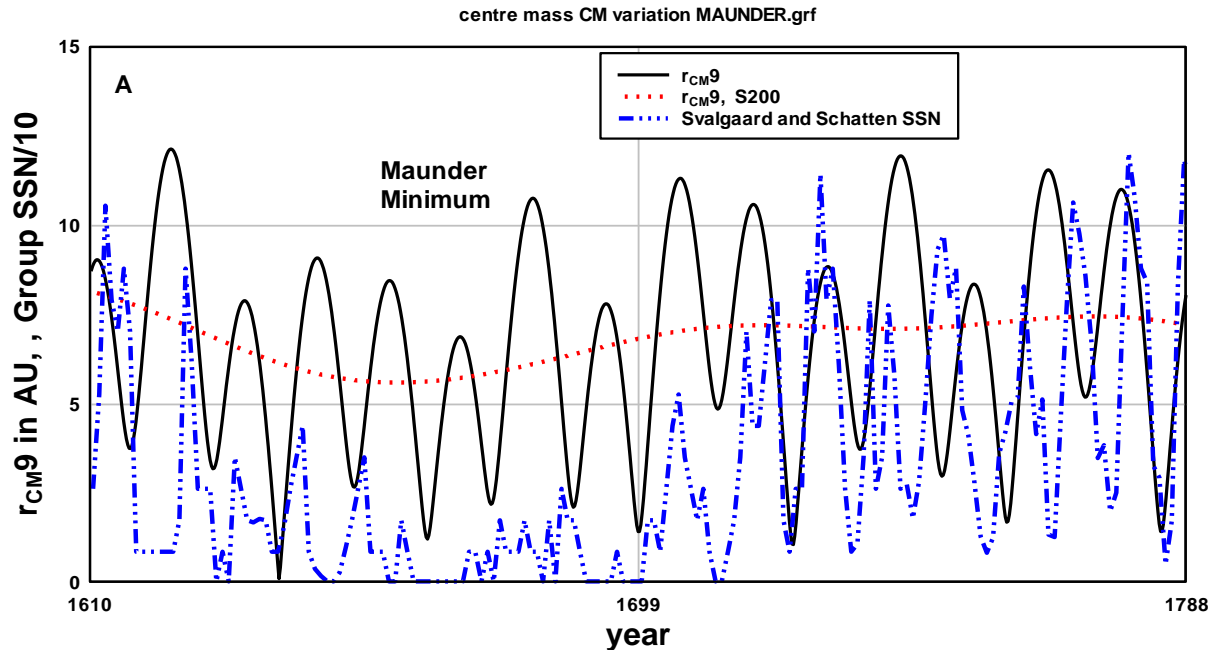
5. Centennial variation in sunspot activity and the dependence on the Sun – barycentre distance.

Centennial variation in solar activity is of particular interest as it covers the time scale associated with the occurrence of grand solar minima and maxima. The importance of predicting the occurrence of grand cycles of solar activity lies with the possibility that grand minima and maxima in solar activity may be associated with significant changes in the global climate, i.e. cooler climate during grand minima and warmer climate during grand maxima, Gray et al (2010).

Here we use the four century long Hoyt and Schatten (1998) reconstruction of sunspot number, (SSN), extending over about thirty five solar cycles, to illustrate the centennial variation in solar

activity. The Hoyt and Schatten (1998) reconstruction has recently been updated by Svalgaard and Schatten (2016) and this is the group sunspot number used here.

Figures 4A, 4B, and 4C compare the reconstructed SSN with planetary centre of mass distance, r_{CM9} , for three intervals each of length 178 years. The interval 178 years was used as solar barycentric motion duplicates itself, very closely, over intervals of 178 years, Jose (1965).



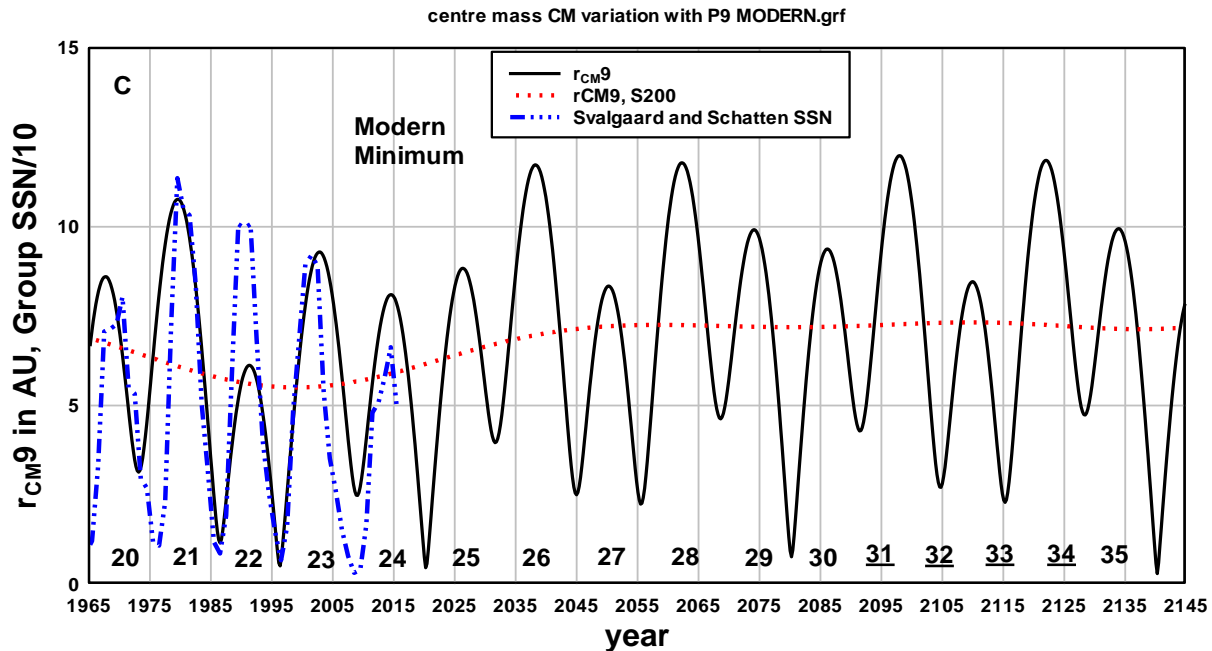


Figure 4. A, B and C. The r_{CM9} variation is calculated at 80 day intervals so the running average over 200 points is equivalent to an average over 16,000 days or roughly four decadal solar cycles. Clearly, there is a decrease in the centennial scale time average of Sun to planet centre of mass distance, r_{CM9} , that closely coincides with the Maunder, Dalton and Modern, grand solar minima in sunspot number. <https://svalgaard.leif.org/research/gn-data.htm>

There are several features evident in Figures 4A, 4B and 4C.

- (1). The r_{CM9} variation in each 178 year long interval is similar, but not exactly similar, from one 178 year interval to the next.
- (2). The 44 year running average of the r_{CM9} variation, the red dotted curve in the Figures, shows a distinct decrease at times corresponding closely to the long term decreases in the Svalgaard and Schatten reconstruction of sunspot number that correspond to the Maunder, Dalton and Modern grand minima in solar activity. We show in Figure 5A and 5B that when the same analysis is applied to the eight planet system the running average of r_{CM8} shows no indication of a decrease during the periods of decreased sunspot number, i.e. the long term running average of r_{CM8} at the centennial time scale is essentially constant.

This observation strongly supports the hypothesis that it is essential to include Planet 9 in barycentric dynamics if it is used as a predictor of centennial scale solar activity.

- (3). The 178 year long intervals in Figures 4 A, B, and C are taken from the beginning of each grand solar minimum. During the first two thirds of each 178 year interval the time of occurrence of decadal maxima in r_{CM9} correspond closely with the time of occurrence of decadal maxima in SSN. This suggests that, during the first two thirds of each 178 year interval,

the average period of the SSN variation is close to the average period of the r_{CM9} variation, ~ 11.9 years, see the spectral content of r_{CM9} in Figure 6. However, in the last one third of each 178 year interval the SSN maxima fall out of synchrony with the r_{CM9} maxima. Examination of Figures 4A and 4B indicate that in the last one third of each 178 year interval the average period between SSN maxima decreased sufficiently that an additional SSN maximum occurred over and above the number of maxima in r_{CM9} in the same interval. Thus, while five maxima in r_{CM9} occurred in this interval, six maxima in SSN number occurred. This suggests that during this latter one third of the interval between grand minima the periodicity of SSN is approximately $5/6$ the periodicity during the first two thirds of each 178 year interval between grand minima, i.e. $(5/6)11.9 = 9.9$ years. This observation further suggests that the average period of SSN cycles overall during the 178 year intervals is $(11.9 \times 0.666 + 9.9 \times 0.333) = 11.2$ years, a period which is very close to the observed average long term decadal periodicity, 10.8 ± 1.9 years, Usoskin et al (2021).

It is evident, from Figures 4A, 4B and 4C that, during the first two thirds of the 178 year interval in each graph, there is no correlation between the amplitude of the r_{CM9} variation and the amplitude of the corresponding solar activity cycle. However, the synchrony of the r_{CM9} cycle and the SSN cycle is evident. Therefore, some very general predictions of solar activity in this time range can be made. Grand solar minima can be expected at intervals of about 178 years and, during the first two thirds of each interval the Schwabe solar cycles will occur synchronously with the cycles in r_{CM9} . Thus, for example, from the variation in r_{CM9} in Figure 4C it is possible to predict a solar cycle 25 maximum at 2026.4, a solar cycle 26 maximum at 2038.2, a solar cycle 27 maximum at 2050.2 and a solar cycle 28 maximum at 2062.3. In some respects this is an improvement on current solar cycle prediction methods which appear to be limited to predicting times and amplitude of solar cycle maxima only one cycle ahead, e.g. (Blowmik and Nandy 2019, Hathaway and Upton 2016, Jiang et al. 2018). Predicting time of solar cycle maximum one cycle ahead should not be very challenging given that the average time between solar cycles 11 ± 1 years. Predicting solar cycle amplitude is evidently much more difficult so that, despite the complicated mathematics involved, the predicted amplitudes vary widely, c.f. the predicted amplitude of solar cycle 25 by the work referenced in this paragraph. Also, note that the predictions by various groups for the peak sunspot number of solar cycle 24 ranged by a factor of 5, between 40 and 200, reflecting the lack of a reliable method (Tobias et al., 2006, Usoskin 2012).

When r_{CM8} is plotted versus SSN for the same intervals, Figure 5A and Figure 5B, it is evident that: (1). The correlation with SSN is much poorer than in the case of r_{CM9} . Primarily, this is due to the fact that r_{CM8} does not show any significant change in average variation over the centennial time range. In addition there is a wide divergence of the \sim decadal periodicity of r_{CM8} , 12.7 years, (see Figure 6), from the observed average periodicity of SSN, ~ 10.8 years.

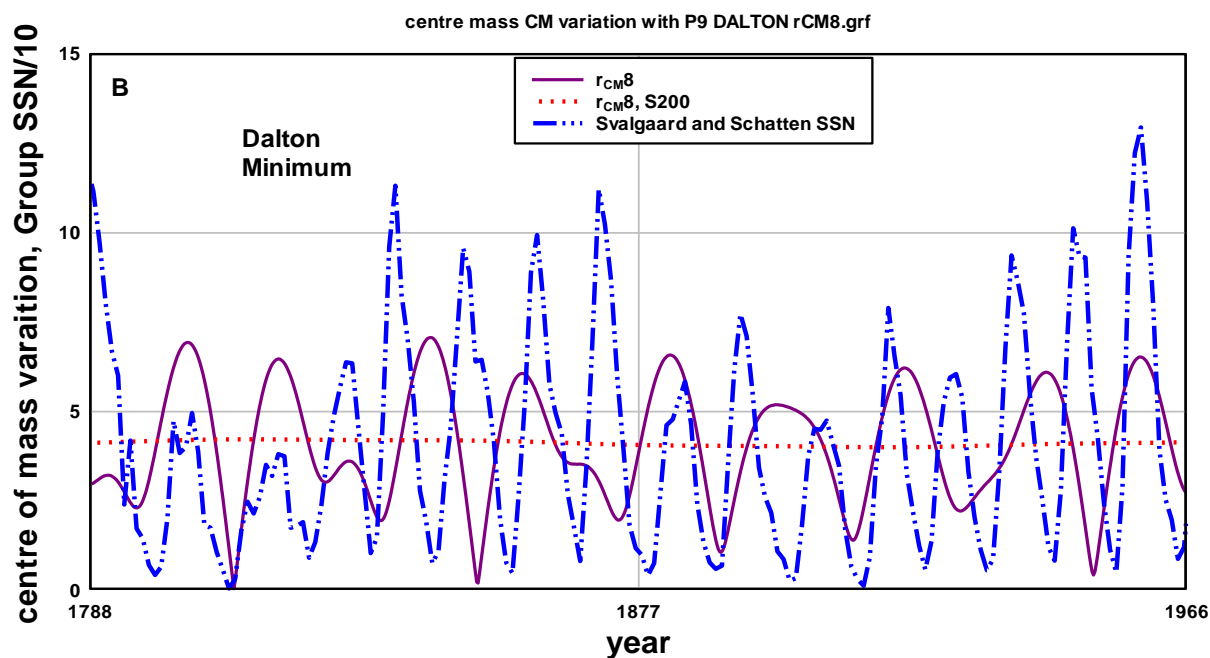
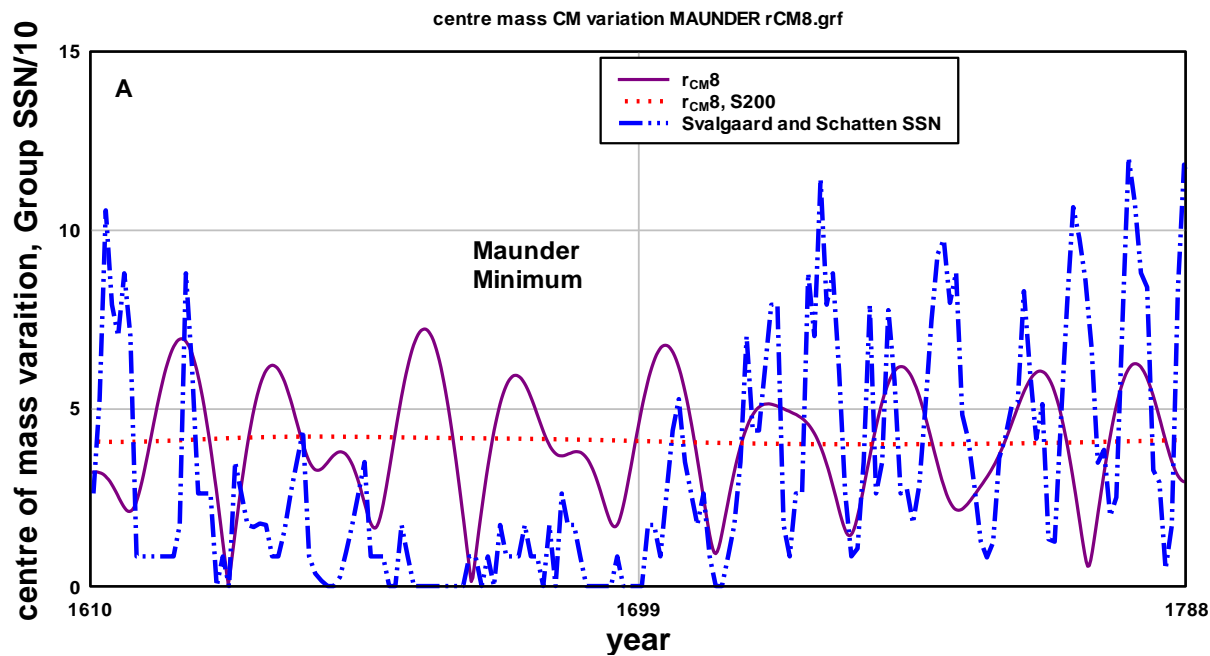


Figure 5. A and B show there is no significant decrease in the centennial scale time average of Sun to planet centre of mass distance, r_{CM8} , coincident with the Maunder and Dalton grand solar minima in sunspot number. There is no centennial scale time variation in barycentric quantities calculated using the eight planet solar system.

It is interesting to compare the spectral content of the r_{CM9} and r_{CM8} variations in the centennial time range, with the spectral content of reconstructed SSN in the same time range, Figure 6.

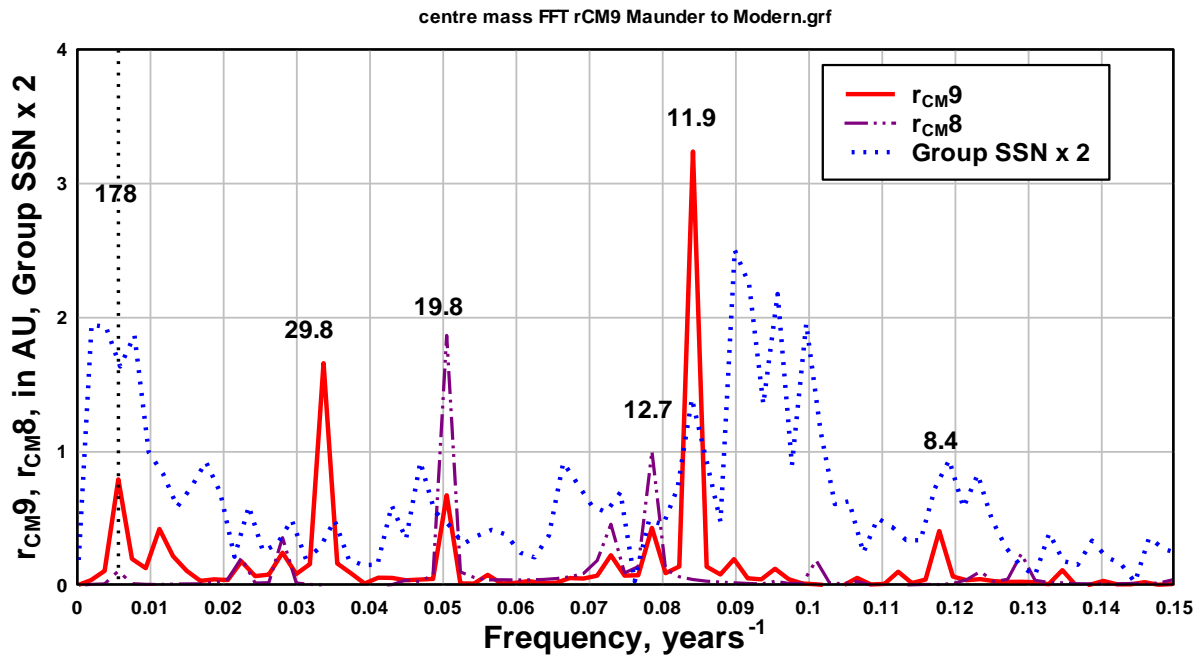


Figure 6. Compares the spectral content of the Sun to planet centre of mass distances r_{CM9} and r_{CM8} with the spectral content of the Svalgaard and Schatten (2016) reconstructed sunspot number. Notice that the dominant periodicity of r_{CM9} , 11.9 years, is close to the average periodicity of sunspot number, 10.8 years. Conversely, the dominant periodicity in r_{CM8} occurs at 19.8 years. Note also that the long term component of r_{CM8} at about 178 years is insignificant in comparison to the r_{CM9} component at ~ 178 years.

Evidently, the reconstructed SSN spectrum and the r_{CM9} spectrum share a significant component at about 178 year period. In contrast the 178 year component in r_{CM8} is insignificant. The r_{CM9} spectrum exhibits peaks at 29.8 years, 19.8 years, 11.9 years, and 8.4 years, with the strongest peak at 11.9 years. The strong peak at 11.9 years is close to the strongest peak, at 0.09 yr^{-1} , 11.1 years, in the SSN variation. In contrast the spectrum of r_{CM8} exhibits a strong peak at 19.8 years and a moderate peak at 12.7 years. This observation supports the hypothesis that barycentric dynamics with Planet 9 included is a better predictor of solar activity at the decadal time scale and centennial time scale than is barycentric dynamics based on the eight planet system.

We complete this section by comparing the reconstructed solar activity over the last 1600 years, Wu et al (2018) available at https://www2.mps.mpg.de/projects/sun-climate/data/SN_composite.txt, with the predicted centennial scale variation of r_{CM9} calculated from barycentric dynamics including Planet 9, Figure 7.

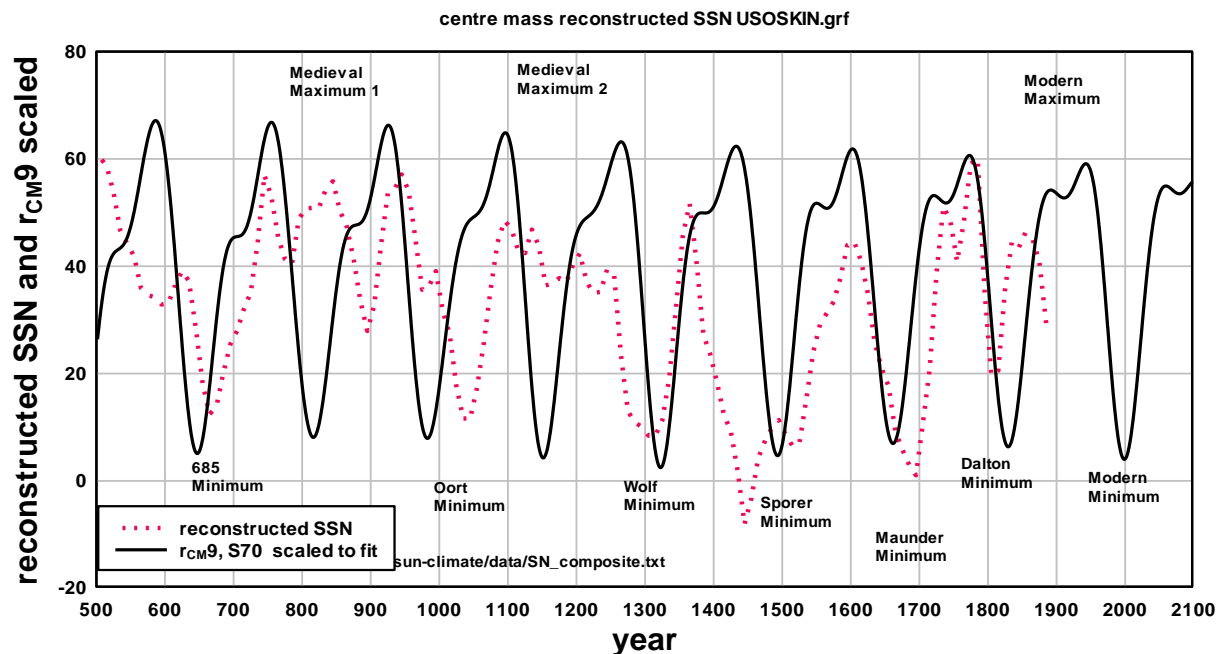


Figure 7. Compares the time variation of the centennial scale time average of Sun to planet centre of mass distance, r_{CM9} , with the reconstructed sunspot number, Wu et al (2018) over the last 1600 years. Seven out of the nine minima in r_{CM9} correspond fairly closely with the grand minima in reconstructed sunspot number when the Modern Grand Minimum is included. This suggests that r_{CM9} is a moderately successful predictor of grand solar minima but that other factors are also in play.

Figure 7 shows that during the last 1600 years there are nine grand minima, including the Modern Minimum, in the centennial scale time average of Sun to planetary centre of mass distance, r_{CM9} . There are seven grand minima in the reconstructed sunspot number, where, in addition to the grand minima listed by Usoskin et al (2007), we specify here the Dalton Minimum and the Modern Minimum. Seven of the grand minima in r_{CM9} are in reasonably close correspondence with seven grand minima in sunspot number, including the Dalton and Modern grand minima. However, there are two instances of grand minima in r_{CM9} that coincide with extended maxima in the sunspot number record, during Medieval Maximum 1 and Medieval Maximum 2. It is therefore evident that the long term variation in r_{CM9} is not a completely reliable predictor of grand solar minima and some effect in addition to planetary influence is involved in determining the occurrence of solar grand minima.

6. Millennial variation in sunspot activity and dependence on the Sun – barycentre distance.

The variability of solar activity over timescales longer than several centuries can only be studied by proxy records of solar activity derived from measurements of the concentration of the cosmogenic isotopes, radiocarbon ^{14}C in tree trunks and ^{10}Be in polar ice, that accumulated due

to the effect of cosmic rays, (Inceoglu et al 2015, Usoskin et al 2016, Viaggi 2021). Due to the known inverse relationship between cosmic rays and solar activity it is possible to reconstruct the variation of sunspot numbers over the approximately 9,000 year Holocene interval, -7000 BC to 2000 AD, Usoskin et al (2016). Spectral analysis of the resulting reconstruction shows a near linear positive trend in the ^{10}Be reconstruction and a long period cyclic component in the ^{14}C reconstruction with a minimum at about 7000 BC, a maximum at 2500 BC and a minimum about 2000 AD. In addition there is a shorter term variation, nominally the Hallstatt cycle, of periodicity about 2400 years. Usoskin et al (2016) conclude that the trend in the ^{10}Be reconstruction and the long period variation in the ^{14}C reconstruction are due primarily to terrestrial processes and remove the variations to provide corrected sunspot number reconstructions, (Figure 5, Usoskin et al 2016). However, both reconstructions retain the common signature of the ~ 2400 year Hallstatt cycle that Usoskin et al. (2016) conclude is most likely a property of long term solar activity. They show, by superposed epoch analysis that grand minima and grand maxima in sunspot number tend to cluster, respectively, at minima and maxima of the ~ 2400 year cycle. Viaggi (2021) provides a very useful summary of the long term periodicity in total solar irradiance as observed by Vieira et al (2011) and Steinhilber et al (2012) and in sunspot number as observed by Wu et al (2018). There are significant differences in the periodicities summarised by Viaggi (2021). The Hallstatt, ~ 2400 year period, cycle is evident only in total solar irradiance and a ~ 1500 year period cycle is evident only in sunspot number. Common to the three studies are the 358, 210, 150 and 88 year periodicities.

In this section we show that barycentric dynamics, including Planet 9, predicts the Hallstaatt ~ 2400 year cycle and the Gleissberg ~ 88 year cycle, whereas barycentric dynamics, excluding Planet 9, shows neither of these long term components of reconstructed solar activity. Figure 8 compares the $r_{\text{CM}8}$ and $r_{\text{CM}9}$ variations from year -8000, (8000 BC) to year +8000, (8000 AD). The points are calculated at 2400 day intervals so that $r_{\text{CM}9}$, S5 corresponds to a running average or smoothing over a 12,000 day, 33 year interval.

The $r_{\text{CM}8}$ variation shows an approximately 800 year period variation that amplitude modulates a cycle of about 0.2 AU in amplitude that repeats at approximately 172 year period. The $r_{\text{CM}9}$ variation shows an approximately 2600 year period cycle, the dotted curve in Figure 8, imposed

on a cycle of about 1.5 AU in amplitude that repeats at approximately 168 year period.

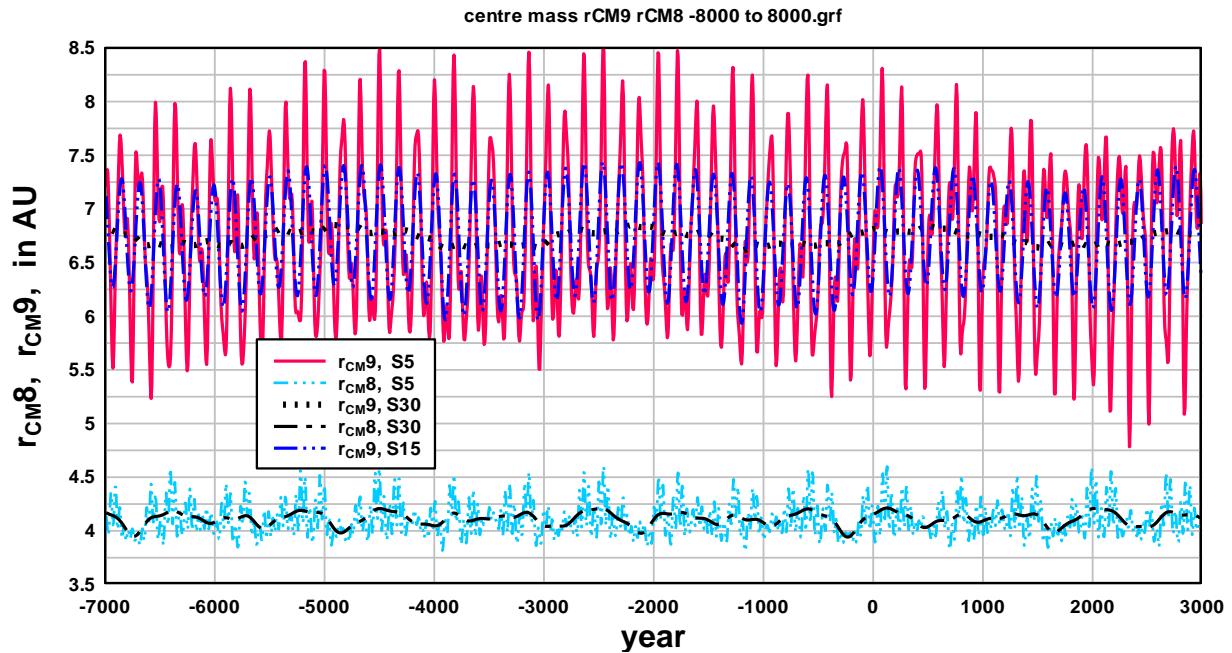
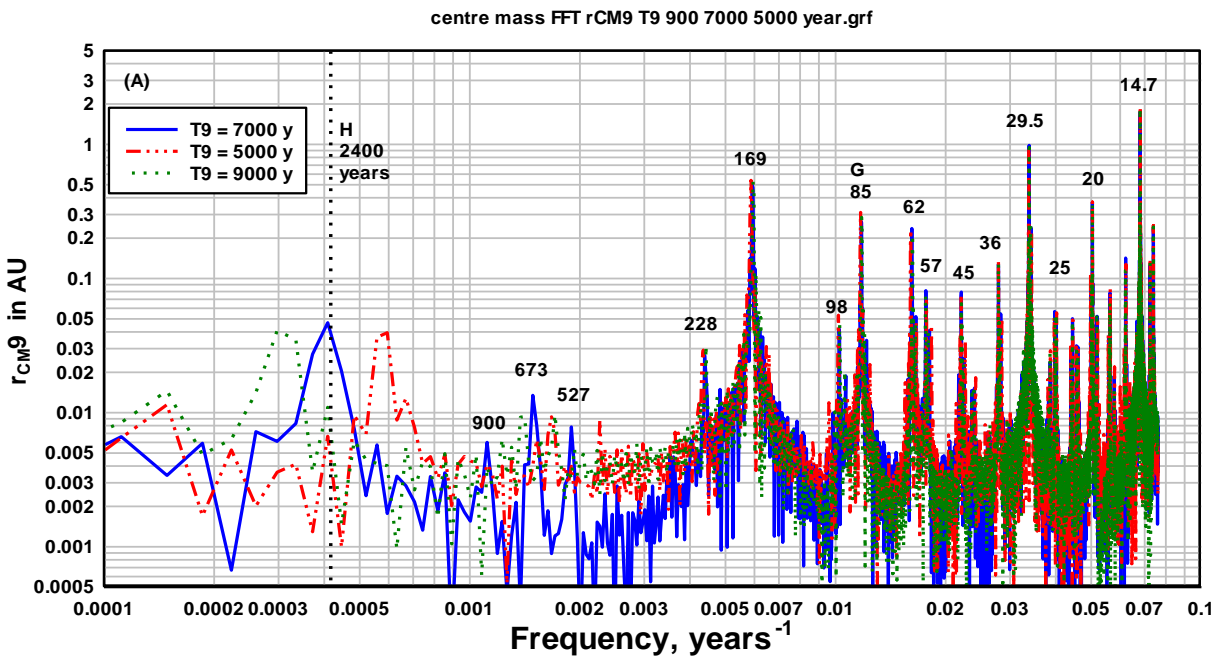


Figure 8. A comparison of the millennium time scale variations of Sun to planet centre of mass distances, r_{CM9} and r_{CM8} . The data points are calculated at 2400 day intervals so that smoothing over 5 data points as with r_{CM9} , S5 corresponds to a running average over a 12,000 day, 33 year interval. The salient features are that an ~ 2600 year cycle modulates an ~ 169 year cycle in r_{CM9} and an ~ 800 year cycle modulates an ~ 170 year cycle in r_{CM8} .

Also in evidence is a broad maximum of r_{CM9} peaks centred on -3000 years with broad minima in r_{CM9} peaks centred on -8000 and on +2000 years. This suggests a long term cycle of about 10,000 year period. However, this long term variation in the peakiness of r_{CM9} is not reflected in the long term average, shown as the dotted line in Figure 8. The absence of a long period cycle in the average occurs as the strong peaks in r_{CM9} are accompanied by broader troughs such that the very long term average in r_{CM9} does not change. This is expected as, in the analysis here, the orbits of all planets, including Planet 9, are circular about the Sun as centre. The indirect evidence, Brown and Batygin (2021), for Planet 9, however, does indicate a significantly elliptical orbit. An elliptical orbit would result in a millennium length variation in r_{CM9} corresponding to the period of Planet 9's elliptical orbit about the Sun. With a strongly elliptical orbit for Planet 9, the long term average value of r_{CM9} , and hypothetically, solar activity would be maximised at Planet 9 aphelion. This would result, according to the hypothesis of this article, in a long period cycle, of about 7000 year period, in solar activity. There are suggestions of such a long period variation in solar activity in the literature e.g. Xapsos and Burke (2009). It is outside the scope of this article to investigate further the effect of an elliptical orbit for Planet 9.

We now compare the spectral content of the r_{CM8} and r_{CM9} variations discussed above and illustrated in Figure 8. The low frequency spectral components in the r_{CM9} variation depend quite strongly on the orbital period of Planet 9 whereas the higher frequency components are relatively independent of the Planet 9 and depend primarily on the orbital periods of the known outer planets, Jupiter, Saturn, Neptune and Uranus. This is illustrated in Figure 9A where the orbital period of Planet 9 is used to tune the Hallstaatt cycle in the r_{CM9} variation to the value of the Hallstaatt period observed by other studies of the reconstructed sunspot number, approximately 2400 years, Usoskin et al (2016). Figure 9A shows the result of the tuning. As the orbital period of Planet 9 is varied from 5000 to 9000 years the Hallstaatt period in the r_{CM9} variation increase from about 1750 to 3200 years and is close to 2400 years when the orbital period of Planet 9 is 7000 years. Notice also that the spectral components of r_{CM9} between 300 and 1000 year period also depend quite strongly on the orbital period of Planet 9. In this range the strongest components when $T_9 = 7000$ years are at 900, 673 and 527 year period.



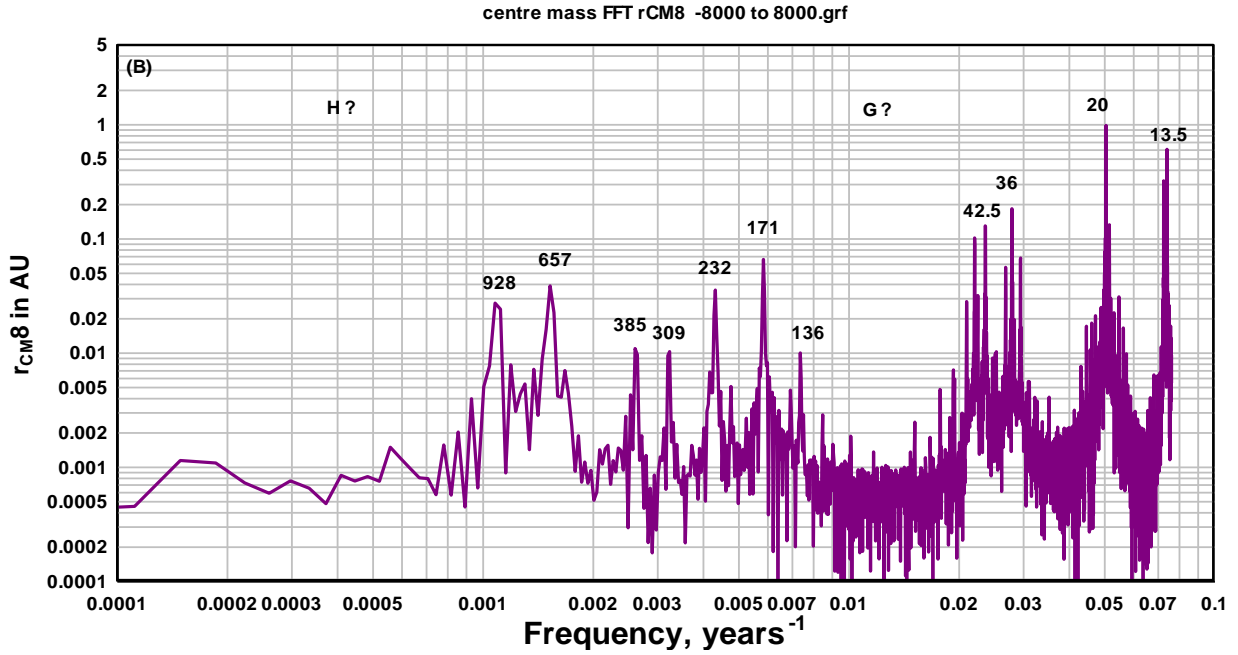


Figure 9. (A), shows the tuning of the long period cycle in the Sun to planet centre of mass distance, r_{CM9} , to the observed Hallstaatt period by varying the orbital period of Planet 9. (B), the much weaker components of r_{CM8} show no evidence of either the Hallstaatt or Gleissberg components.

The ~ 2400 year cycle in r_{CM9} is shown to be reasonably coherent with reconstructed sunspot number over the time range between -7000 years and 2000 years, Figure 10.

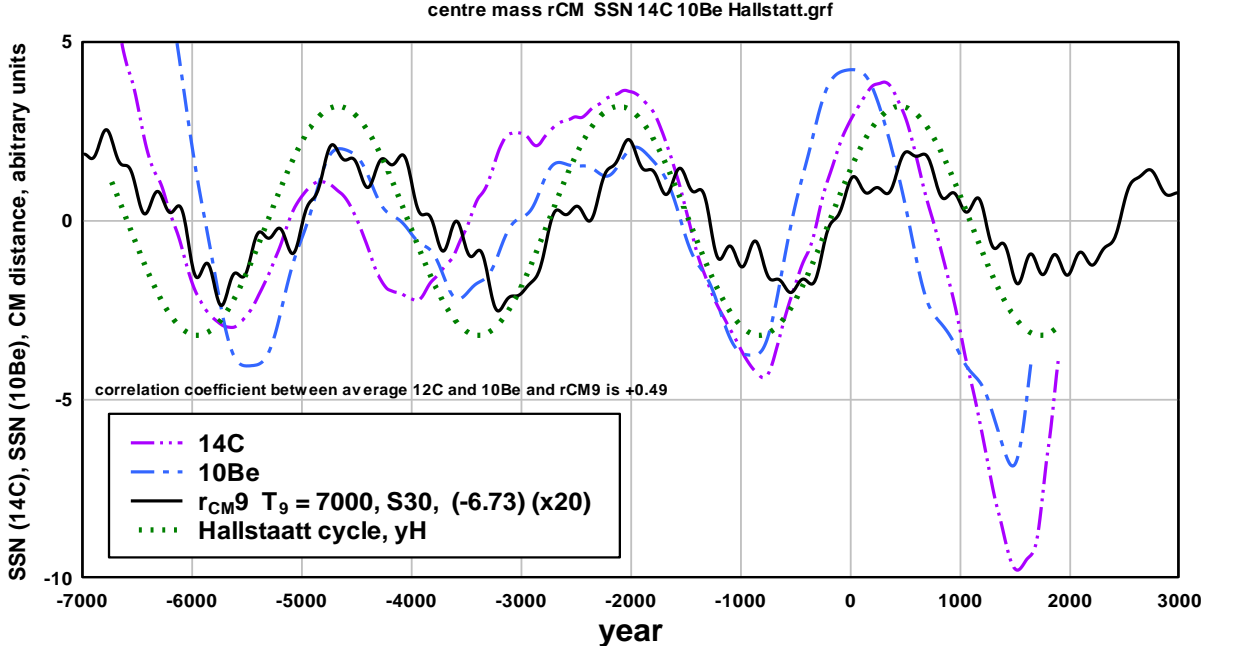


Figure 10. Long term planetary centre of mass distance, r_{CM9} , with $T_9 = 7000$ years, as in Figure 8, after removing the constant term (-6.73 AU) and scaling (by x 20), to fit to the singular spectral analysis component of the reconstructed sunspot number, SSN, from ^{14}C and ^{10}Be data, (Usoskin et al. 2016), courtesy Ilya Usoskin. The

correlation coefficient between the average of the ^{14}C SSN variation and the ^{10}Be SSN variation and the $r_{\text{CM}9}$ variation is +0.49. Also shown is the Hallstaatt cycle variation, y_H , calculated from the parameters of phase modulated sidebands in the spectrum of reconstructed SSN, see Section 7.

The coherence between the Hallstaatt variation in reconstructed sunspot number and the long term cycle derived from solar barycentric dynamics is further evidence that barycentric dynamics with Planet 9 included is a more effective predictor of long term solar activity than barycentric dynamics without Planet 9. It is interesting to note that the Hallstaatt cycle is evident in the basic barycentric variable, $r_{\text{CM}9}$, whereas convoluted approaches are necessary to find evidence of a long term Hallstaatt variation in barycentric dynamics without Planet 9, e.g. (Charatova 2000, Scafetta et al 2016).

7. Transformation of amplitude modulated planetary forcing into phase modulated solar activity.

Knudsen et al (2009) measured the spectral content of the reconstructed sunspot number in the period range between 50 and 500 years, reproduced as Figure 11 below. There are consistently prominent components at 88, 150, 220 and 400 years.

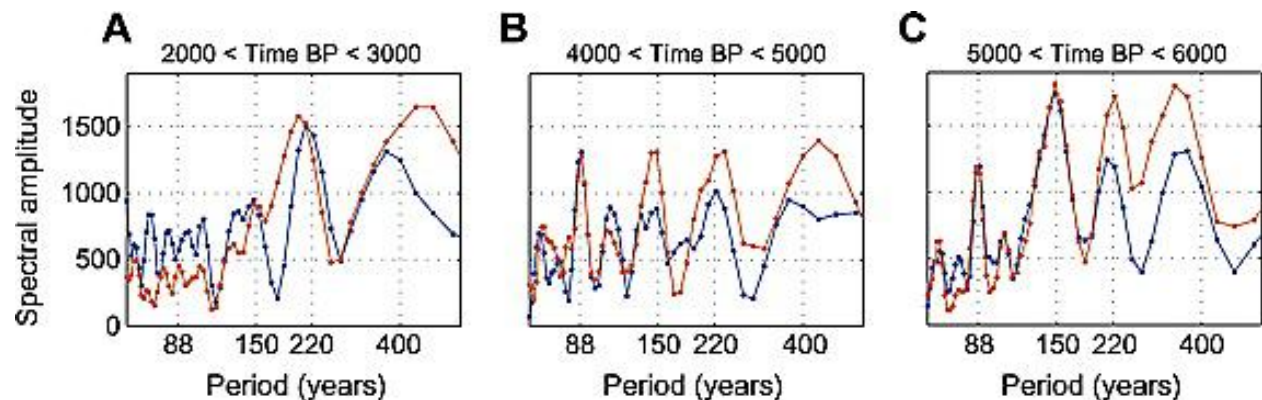


Figure 11. Periodograms of reconstructed sunspot number over successive intervals of time, Knudsen et al (2009), show that the mid range periodicity is dominated by components at 88, 150, 220 and ~ 400 years. The periodicity at the Jose planetary period, ~170 years, is noticeable by its absence.

The most obvious differences between the spectral content of the planetary induced $r_{\text{CM}9}$ variation, Figure 9A, and the measured spectral content of reconstructed SSN as reported by Knudsen et al (2009), Figure 11, is the absence of the prominent 169 year planetary component in the SSN spectra and the absence of the 150 year component, prominent in the SSN spectrum, from the spectrum of the planetary $r_{\text{CM}9}$ variation.

The absence of the 169 year Jose periodicity, the most prominent component in the $r_{\text{CM}9}$ variation in Figure 9A, from the spectra of reconstructed SSN is a serious challenge to the

hypothesis of this article, i.e. that solar barycentre dynamics including Planet 9 is a reliable predictor of solar activity over short and long time scales. However, the absence of a peak at a planetary frequency, f_j , in the SSN spectrum would occur if the component at the Jose frequency f_j is phase modulated during the transformation of planetary forcing into sunspot emergence by a longer term modulating cycle at frequency, f_m . In that case all the power in the component at the planetary frequency, f_j , would, in the SSN spectrum, be split into two sidebands with frequencies, f_1 and f_2 where

$$f_1 = f_j - f_m/2 \quad \text{and} \quad f_2 = f_j + f_m/2. \quad (7)$$

If f_1 and f_2 are known, we have two equations and the two unknowns, f_m and f_j , can be found. With $T_2 = 150$ years and $T_1 = 220$ years we find $T_m = 471$ years and $T_j = 178$ years. As $T_j = 178$ years corresponds very closely to the Jose planetary period the results of Knudsen et al (2009) support the idea that the planetary periodicity is absent from the spectrum of reconstructed sunspot number due to phase modulation of the Jose component by a lower frequency cycle. In this section we demonstrate that the entire low frequency part of the reconstructed SSN spectrum, i.e. the spectrum in the frequency range below 0.015 year^{-1} or in the period range above 67 years devolves from phase modulation of a planetary component at 176.5 year period, nominally the Jose period T_j , and from amplitude modulation of what is, hypothetically, the first harmonic of the Jose component at $T_G = 88.25$ years. This periodicity is usually referred to as the Gleissberg cycle periodicity in solar activity. We show here that the Gleissberg cycle component is amplitude modulated by the same long term cycles that phase modulate the Jose component. In the case of amplitude modulation a fraction of the power of the modulated component is transferred into sidebands spaced equally on either side of the amplitude modulated component, Peristykh and Damon (2003). In this case the sidebands occur at frequencies, f_1 and f_2 , given by

$$f_1 = f_G - f_m \quad \text{and} \quad f_2 = f_G + f_m \quad (8)$$

where f_G is the Gleissberg frequency. We use the multi-isotope composite of sunspot number since year -6755 obtained by Wu et al (2018), and reproduced here as Figure 12. After removing the constant term, a frequency spectrum is obtained using the padded FFT function in the DPlot application. The spectrum is shown in Figure 13, and referred to hereafter as the SSN spectrum.

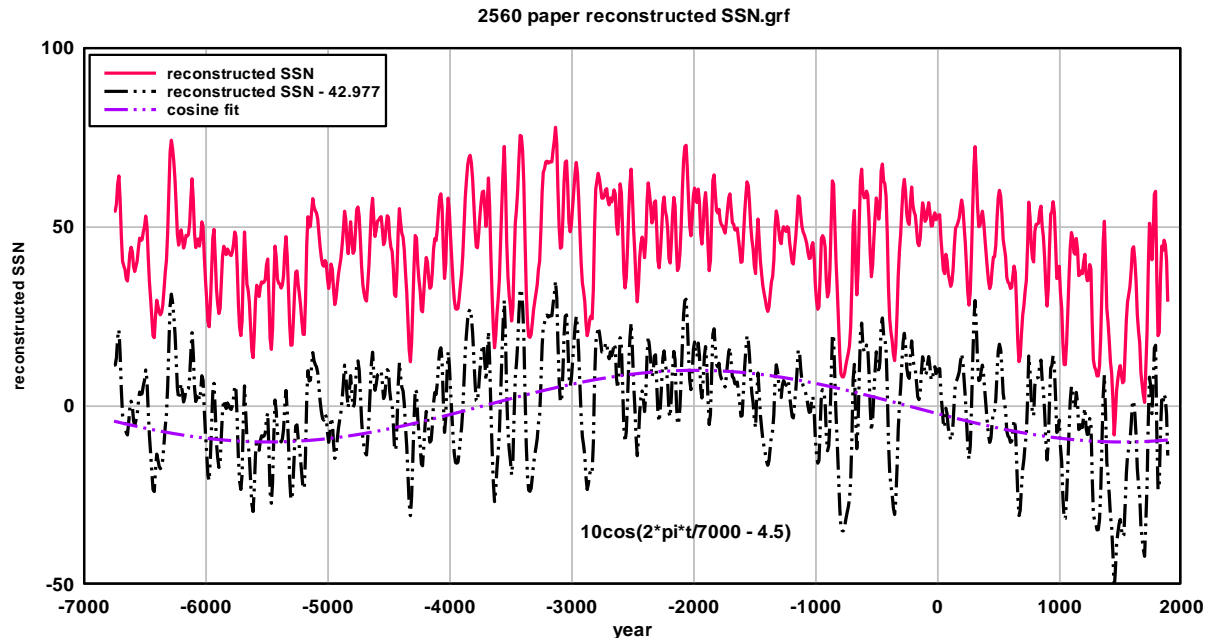


Figure 12. The reconstructed sunspot number between -6755 and 1885, Wu et al (2018). https://www2.mps.mpg.de/projects/sun-climate/data/SN_composite.txt Also shown the the SSN after removing the constant term and fitting a 7000 year period cosine. The frequency spectrum of the reconstructed sunspot number is shown in Figure 13.

Figure 13 shows the frequency spectrum of the reconstructed sunspot number. We are here primarily interested in two components of planetary forcing that are relevant to the reconstructed SSN spectrum.

(1). A planetary component at 176.5 year period, ($0.005666 \text{ year}^{-1}$), associated with the Jose period. This component does not actually appear in the SSN spectrum as phase modulation by longer period components splits this component so that all power shifts into sidebands on either side of the frequency $f_j = 0.005666 \text{ years}^{-1}$. The Jose frequency is marked with a full reference line, labelled 176.5 J, in Figure 13. Sidebands on either side of this reference line are labelled a to e in Figure 13. We note that the two most prominent sidebands are labelled, d. The lower frequency sideband labelled d occurs at period 209 years and the higher frequency sideband labelled d is at period 151 years. The two sidebands correspond to the prominent spectral components at ~ 150 years and ~ 220 years observed by Knudsen et al (2009) in reconstructed SSN, see Figure 11 above. Two of the longer period components, the ~ 2500 year Hallstaatt cycle and the ~ 1070 year Eddy cycle, are also indicated by reference lines in Figure 13.

(2). The other component of interest is the component at 88.2 years, ($0.01134 \text{ years}^{-1}$) usually associated with the Gleissberg cycle. This component is clearly prominent in the SSN spectrum and is also prominent in the r_{CM9} planetary spectrum, Figure 9A, where it occurs at ~ 85 years.

However, it is evident that a significant fraction of the power in this component is shifted by amplitude modulation by longer period components into sidebands. The amplitude modulation of the Gleissberg cycle by longer period components of the SSN variation was discussed in some detail by Peristykh and Damon (2003). In the present paper, for reasons of clarity, only two sets of sidebands of the 88 year Gleissberg cycle are labelled in Figure 13, at b and d.

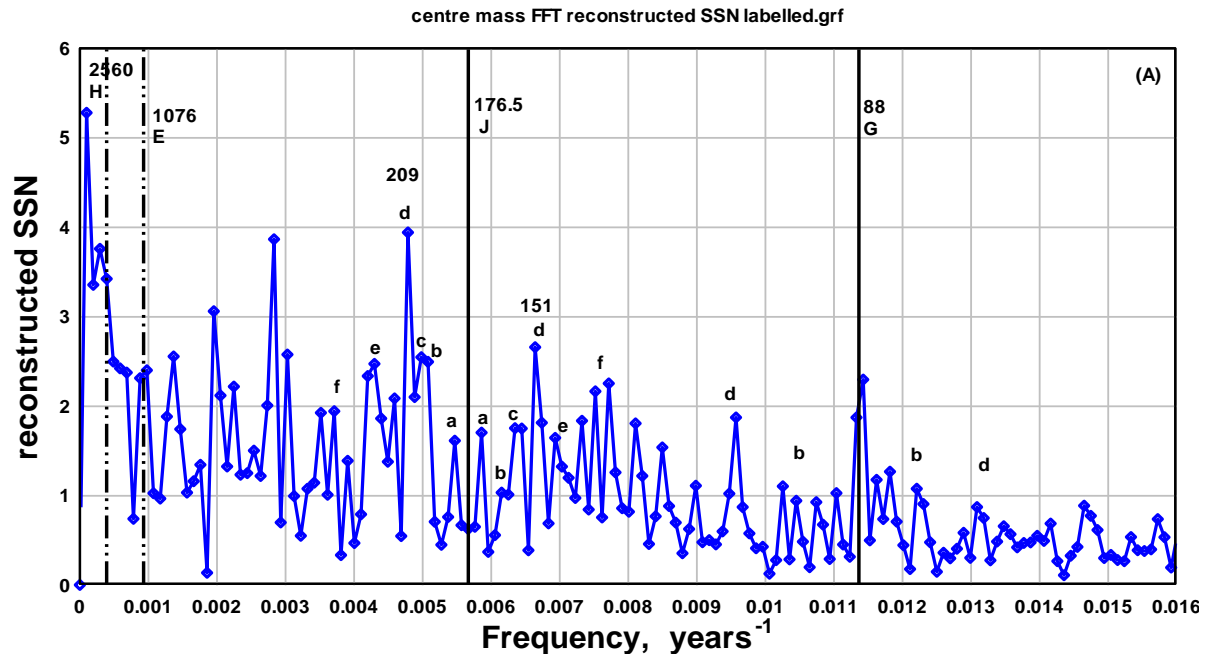
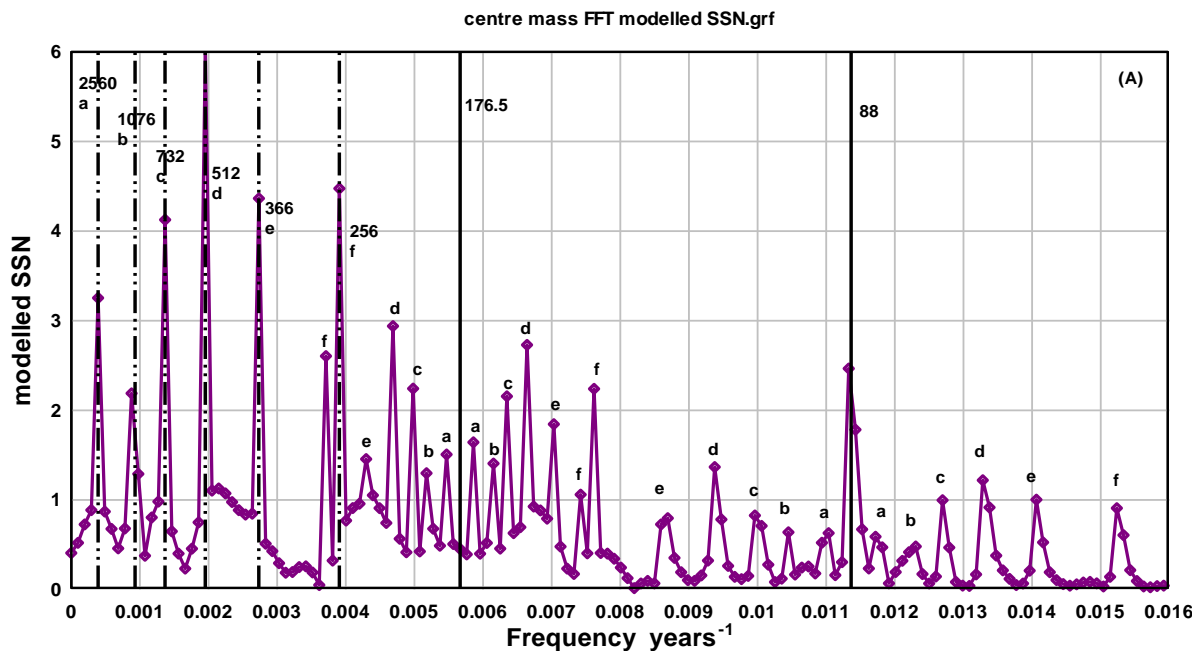


Figure 13. The spectral content of the reconstructed sunspot number, obtained by Fast Fourier Transform. The two full line reference lines indicate the 176.5 Jose periodicity, that is not actually present in the spectrum due to its phase modulation by longer period components, and the 88 year Gleissberg periodicity, that is present as it is amplitude modulated by longer period components. Also shown, with broken reference lines, are the longer period Hallstaatt and Eddy components that are two of the effective modulators of the Jose and Gleissberg components.

We now estimate the frequencies and periods of the lower frequency components in the SSN spectrum. The periods of the lowest frequency, longest period, components are difficult to establish directly as the resolution of the spectrum at the lower frequency end is very limited. However, the resolution in the phase modulation region around the 176.5 year Jose period is much higher. Consequently, the frequencies of the sidebands in this region can be used via equation 7 to estimate the frequency of the lowest frequency phase modulating components. For example, the frequency difference between the two inner sidebands labelled a and a in Figure 13 is, according to equation 7, equal to the frequency, f_m , of the modulating component. The lower frequency is $0.0054687 \text{ years}^{-1}$, the higher frequency is $0.0058594 \text{ years}^{-1}$, and the difference is $0.0003907 \text{ years}^{-1}$. In this way it was possible to estimate the frequency of the Hallstaatt component as $f_m = 0.0003907 \text{ years}^{-1}$ despite the very low resolution at this frequency. The corresponding Hallstaatt period, as estimated in this way is 2560 years. As the

reconstructed sunspot number extends over 10,000 years the resolution in the spectrum is 0.0001 years. Thus, although the spectral peaks appear well defined the observational error in fm must be about 20% and, accordingly the estimated Hallstaatt period is 2560 +/-500 years. By a similar process the frequencies and periods of a total of six modulating components were estimated. The periods were 2560, 1076, 732, 512, 366, and 256 years. The components at these periods, along with the 176.5 year Jose component and the 88 year Gleissberg component, were used with equations 9 to 14 to generate a model spectrum as shown in Figure 14A.

The lower frequency range of this model spectrum, between 0 and 0.008 years⁻¹, comprises the low frequency modulating components labelled a to f, and the sidebands resulting from phase modulation of the 176.5 year period Jose component by the low frequency components. Note that due to the phase modulation the Jose component itself does not appear as a spectral peak. The two groups of sidebands of the Jose component are also labelled a to f. At the higher frequency end of the modelled spectrum, between 0.008 and 0.016 years⁻¹ the 88 year period Gleissberg component along with the sidebands due to amplitude modulation of the Gleissberg component by the low frequency components a to f is shown. Note that, at the lower frequency end of the Jose sidebands, at about 0.004 years⁻¹, the Jose sideband, f, overlaps with the highest frequency modulating component, labelled 256 f. At the higher frequency end of the Jose sidebands, at about 0.0075 years⁻¹, the Jose sideband f overlaps the lowest frequency sideband, f, of the amplitude modulated Gleissberg cycle.



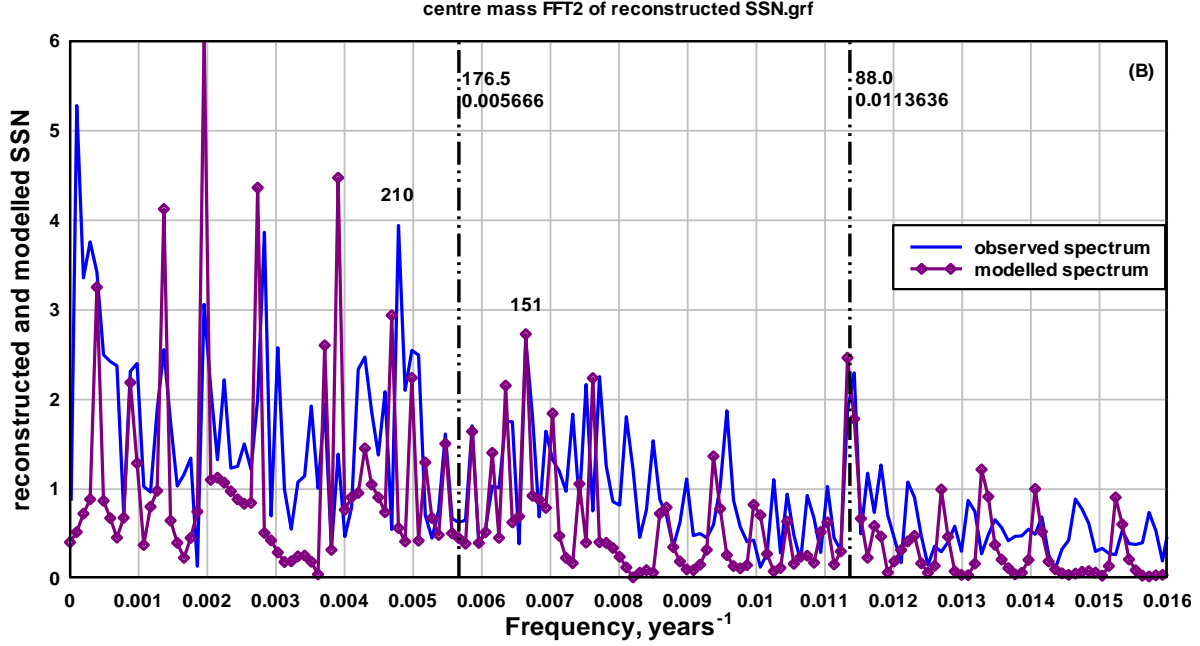


Figure 14. (A) The modelled spectrum of the reconstructed SSN based on equations 9 to 13. The low frequency modulating components are marked, e.g. 512 d. The modulated sideband components are marked a to f for both the phase modulated Jose component and the amplitude modulated Gleissberg component. (B) The modelled spectrum superimposed on the actual spectrum of reconstructed sunspot number.

The angular frequencies of the low frequency modulating components in Figure 14A are

$$\omega_a = 2\pi/2560, \quad \omega_b = 2\pi/1076, \quad \omega_c = 2\pi/732, \quad \omega_d = 2\pi/512, \quad \omega_e = 2\pi/366, \quad \omega_f = 2\pi/256 \text{ rd/yr} \quad (9)$$

The low frequency modulating components are reproduced by the function

$$y_M = 4\cos(\omega_a t) + 3\cos(\omega_b t) + 5\cos(\omega_c t) + 7\cos(\omega_d t) + 5\cos(\omega_e t) + 5\cos(\omega_f t) \quad (10)$$

The angular frequencies of the modulated components are:

$$\text{Jose, } \omega_J = 2\pi/176.5, \quad \text{Gleissberg, } \omega_G = 2\pi/88.2 \quad (11)$$

The phase modulated sideband components centred on the Jose period are given by:

$$y_{PM} = \cos(\omega_J t) [4\cos(\omega_a t/2) + 3\cos(\omega_b t/2) + 5\cos(\omega_c t/2) + 7\cos(\omega_d t/2) + 5\cos(\omega_e t/2) + 5\cos(\omega_f t/2)] \quad (12)$$

The amplitude modulated side band components centred on the Gleissberg period are given by:

$$y_{AM} = \cos(\omega_G t) [7 + 4\cos(\omega_a t) + 3\cos(\omega_b t) + 5\cos(\omega_c t) + 7\cos(\omega_d t) + 5\cos(\omega_e t) + 5\cos(\omega_f t)]/2 \quad (13)$$

The total spectrum, as shown in Figure 14A, is found from the FFT of

$$Y = Y_M + Y_{PM} + Y_{AM} \quad (14)$$

None of the components used in this model have the phase of the component included so that y in equation 14 does not model the actual time variation of the reconstructed SSN. However, the spectrum of y in equation 14 is adequate to model the spectrum of the reconstructed SSN.

The phase of the components can be found quite simply provided the spectral peaks in the FFT of the reconstructed SSN are well defined. To illustrate the procedure we use the two sideband peaks associated with phase modulation of the Jose 176.5 year component by the Halstaatt 2560 year cycle, the peaks labelled a and a in Figure 13 and 14A. The amplitude and phase of the higher and lower frequency components in the reconstructed sunspot number are observed to be

$$a_H = 1.706 \cos(2\pi 0.0058594t - (-1.48137))$$

$$a_L = 1.615 \cos(2\pi 0.0054687t - (-2.60159))$$

When the time variation of the two sideband components are generated and added we obtain $c = a_H + a_L$, the signal due to the phase modulation of the Jose component by the Hallstaatt cycle. The variation of c is shown as the red broken line in Figure 15. Phase modulation of the Jose component by the Hallstaatt cycle occurs when the Jose component time variation is inverted or reversed in phase during each successive positive swing of the Hallstaatt cycle. The actual mechanism by which the inversion associated with phase modulation occurs is outside the scope of this article. However, the inversion can be mathematically accomplished by multiplying the Jose component with a sinusoidal variation at twice the period of the Hallstaatt cycle, i.e. by multiplying the Jose component by a variation of the form

$$y_{H2} = 3.2 \cos(2\pi t / (2 * 2560) - \phi)$$

where, ϕ , is the phase angle is to be determined. The phase angle can be determined by adjusting ϕ so that the variation y_{H2} matches the peaks of the c variation in Figure 15. The result,

$$y_{H2} = 3.2 \cos(2\pi t / (2 * 2560) - 0.55)$$

is shown as the full black line in Figure 15. This variation is the first subharmonic of the Hallstaatt variation so the Hallstaatt variation itself, y_H , is given by

$$y_H = 3.2 \cos(2\pi t / 2560 - 1.1).$$

This is shown as the broken blue curve in Figure 15. This estimate of the Hallstaatt cycle variation is reproduced in Figure 11, labelled “Hallstaatt cycle yH”, to illustrate that this variation is coherent with other estimates of the time variation of the Hallstaatt cycle.

Once the frequency, amplitude, and phase of the Hallstaatt cycle has been determined from the parameters of the two spectral components, a_H and a_L , in the reconstructed SSN spectrum, it remains to determine the phase of the 176.5 year, Jose component. Due to phase modulation, the Jose component does not appear in the spectrum of the reconstructed SSN so the frequency and phase angle of this component must be found indirectly. Finding the frequency is straightforward. It is the mid frequency between any of the sideband pairs. The phase angle is found as follows. The variation, $c = a_H + a_L$, derived from the two sidebands labelled “a” and shown as the broken red line in Figure 15 is due to phase modulation of the Jose component by the Hallstaatt cycle and is generated by a function of the form

$$yy = 3.2 \cos(2\pi t/176.5 + \phi_J) \cos(2\pi t/(2*2560) - 0.55)$$

where ϕ_J is the phase angle of the Jose component, yet to be determined. The phase angle ϕ_J can be determined by adjusting ϕ_J in the function yy so that the function yy fits the variation $c = a_H + a_L$ in Figure 15. The result is $\phi_J = 2.0$ radians and

$$yy = 3.2 \cos(2\pi t/176.5 + 2.0) \cos(2\pi t/(2*2560) - 0.55)$$

This variation is shown as the green broken line in Figure 15. The variation has been displaced by - 0.5 units for additional clarity. Clearly, the extended analysis that derives the Hallstaatt cycle and Jose cycle phase angles from the two components, a_H and a_L , in the spectrum of reconstructed sunspot number has a large error, probably of the order 20%. For example the Jose phase angle would be expressed as $\phi_J = 2.0 \pm 0.4$ radians or $\phi_J = 115^\circ \pm 25^\circ$. However, a detailed error analysis is outside the scope of this paper.

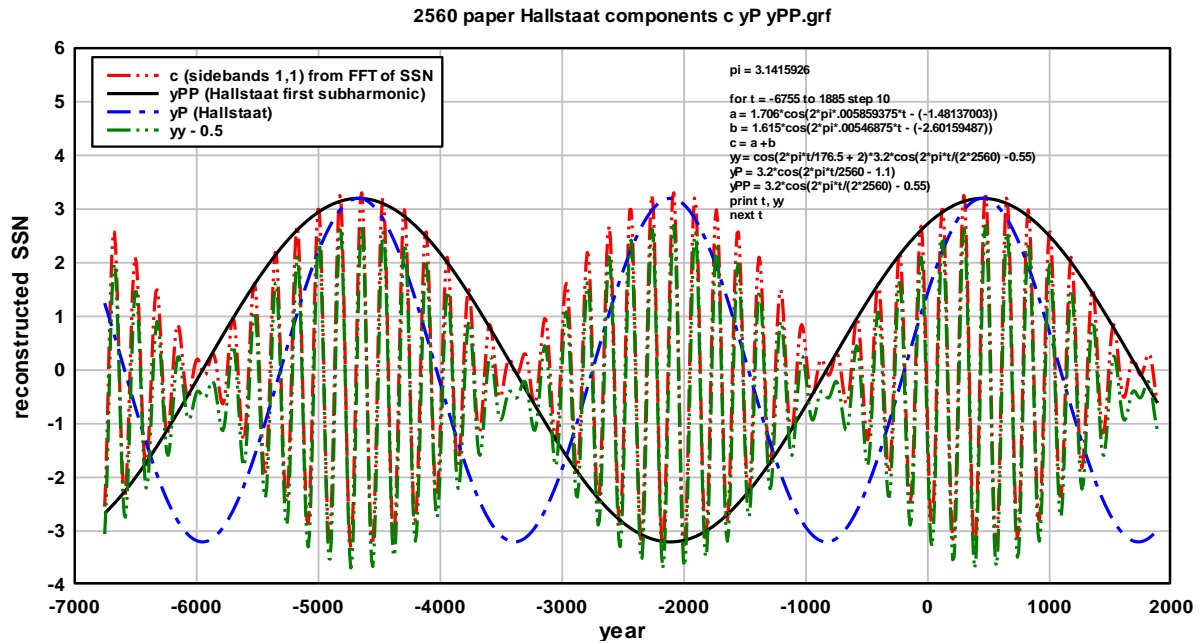


Figure 15. Procedure to find the phase angles of the Hallstaatt and Jose components in the reconstructed sunspot number. The time variation from the two innermost sidebands, a_H and a_L is found, $c = a_H + a_L$, shown as the red broken line. This variation is phase reversed or inverted at each cycle of the Halstaatt modulating component. The negative part of the first sub harmonic of the Halstaatt variation, y_{PP} , corresponds to this inversion. The inversion by the function y_{PP} is represented by the full black curve. The phase of y_{PP} , adjusted to fit the c variation, is 0.55 radians. It follows that the Hallstaatt variation itself, y_P , is represented by the broken blue line with phase angle 1.10 radians. The phase modulation of the Jose component by the Hallstaatt component is represented by the broken green line, (displaced by -0.5 units for clarity) where the phase of the Jose component, 2.0 radians, was adjusted to fit the phase modulated Jose variation, broken green curve, to the variation, C , derived from the sideband parameters, broken red curve.

It is interesting, for completeness, to compare the phase angle of the Jose component in the spectrum of the r_{CM9} variation, Figure 9A, that was found from the FFT spectrum to be $\phi_j = -59^\circ$, with the estimate of ϕ_j from the spectrum of reconstructed sunspot number, $\phi_j = 115^\circ \pm 25^\circ$. The two estimates obviously differ. However, the difference between the two is $174^\circ \pm 25^\circ$, suggesting an inversion of 180° may be involved. In principle, each of the phase modulated sideband pairs, labelled a to e in Figures 13 and 14A can be treated in the way outlined in this Section to obtain the time variations of the phase modulated components and the time variations of all the five modulating components. Similarly, the amplitude modulated sideband pairs centred on the Gleissberg frequency can be used to obtain the time variations of the amplitude modulated components and a second estimate of the time variations of the modulating components. In practice this procedure would provide only an approximation of the actual time variations as most of the sidebands are not as clearly defined in the spectrum as the two sidebands, a_L and a_H , used in the above example. This lack of definition is due in part to the lack of resolution of the standard FFT used for spectral analysis, the fact that the

reconstructed SSN is affected by noise, and the fact that there is considerable interference between the sideband components.

Another interesting aspect of this analysis is that the long term modulating components at periods 256, 366, 512, 732, 1076 and 2560 years appear to be sub harmonics of the Jose component at period 176.5 years. Figure 16 illustrates this point. In Figure 16 the measured long term periods as determined from the frequency spacing between the phase modulated sidebands in Figure 13 and 14A are plotted against multiples of 176.5 years. This shows that the periods of the observed long term components in the reconstructed SSN are very close to sub harmonic periods of the Jose, 176.5 year period. Given that the ~88 year Gleissberg component can be associated with the first harmonic of the 176.5 year Jose component this suggests the rather intriguing idea that all of the millennium time scale frequency components of the reconstructed sunspot number could devolve, via sub harmonics and harmonics and phase and amplitude modulation, from a single controlling periodic component, the 176.5 year Jose component. Demonstrating the correctness of this concept would be a challenging future project. However, the main objective of this section is to illustrate why the dominant spectral component in the planetary forcing, shown in Figure 9A at ~ 169 years, does not appear in the spectra of the reconstructed SSN. Although the concept of long term solar activity devolving entirely from the Jose periodicity is intriguing further work on this is outside the scope of this article.

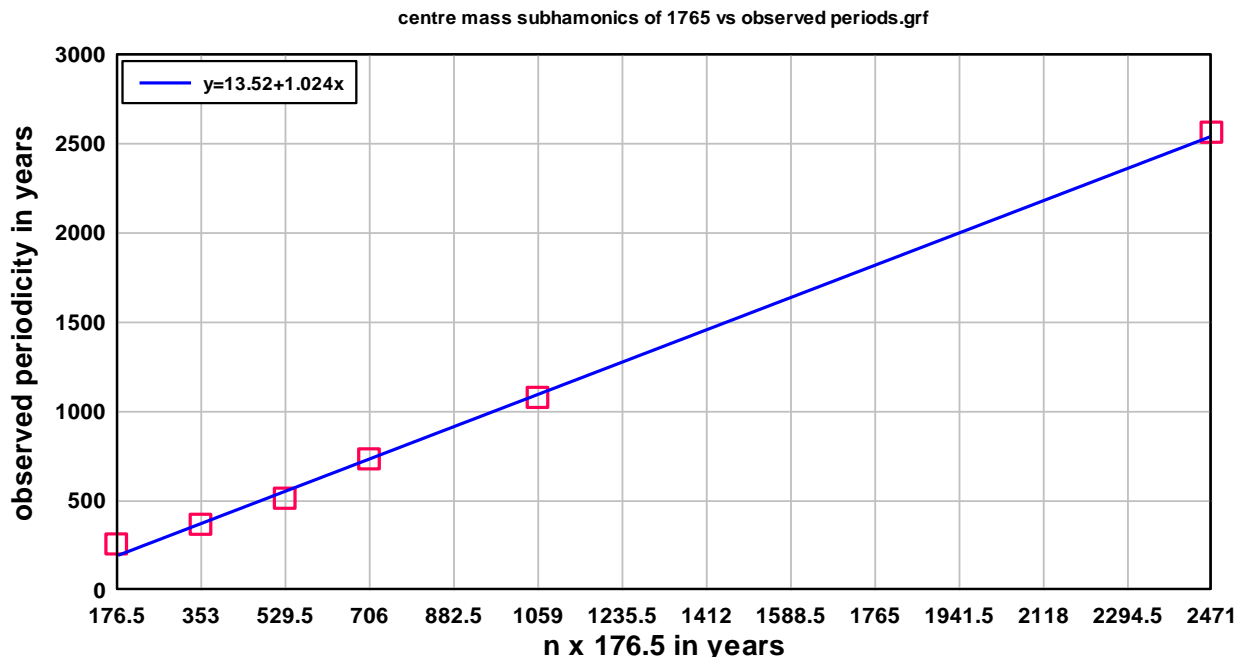


Figure 16. Shows that periods of the long term components of the reconstructed sunspot number record, Wu et al (2018), obtained by analysis of sidebands in the FFT, periods 256, 366, 512, 732, 1076, and 2560 years, are close to sub harmonics of the Jose period, 176.5 years.

8. Relevance to locating Planet 9

The hypothesis of this paper is that including Planet 9 in the calculation of solar barycentric dynamics provides for improved prediction of the time variation of solar activity. The corollary is that, if Planet 9 improves the predictive capability of barycentric dynamics this, in itself, provides additional indirect support of the existence of Planet 9. The general findings of this article do indicate improved prediction of solar activity with the inclusion of Planet 9 and as a result support the corollary.

Indirect evidences of the location of Planet 9 arising from the article are, firstly, an indication of heliographic longitude of Planet 9. This was found in section 4 where the heliographic longitude of Planet 9 was adjusted to fit the r_{CM9} variation to the decadal solar activity variation during solar cycles 20 to 24, Figure 1A. The result was expressed as: $HGIL9 = 50 \pm 10$ degrees on January 01, 1965. A second finding was that the Planet 9 orbital period could be tuned to the observed Hallstaatt cycle period with the finding that the Planet 9 orbital period was close to 7000 years, Section 7, Figure 9A.

These findings are important with respect to the hypothesis of Planet 9 because there is currently no information about the position of Planet 9 in its orbit, Bailey et al (2018). This is because the astronomical information about Planet 9 derives from the anomalous orbits of Kuiper Belt objects and, as the orbits have been fixed for millions of years, the orbits provide no dynamic or short term information about the current position of Planet 9. With an estimated orbital period of 7000 years, and with the approximation of a circular orbit, the heliographic longitude of Planet 9 advances by 0.051 degrees per year. So, the two findings provide that the heliographic longitude has increased from 50° in 1965 to 53° in 2022.

9. Discussion and conclusion

As mentioned in the introduction the consensus approach to understanding the variability of solar activity is based on the solar dynamo being forced by stochastic processes interior to the Sun or by the solar dynamo occasionally changing from regular to chaotic cycling, Charbonneau (2010). There appears to be no prospect of predicting future solar activity with either approach, so the theories are not testable by the usual scientific method of prediction followed by observation. The alternative approach to understanding solar variability is to assume processes interior to the Sun are influenced by planet motion, either by direct gravitational effect of the planets on the Sun or via the periodic wobbles about the solar system centre of mass imposed on the Sun as the planets orbit. Both approaches hold the promise of predicting both short and long term solar activity but neither approach has, to date, provided useful predictive capability. The hypothesis of this article is that the inclusion of Planet 9 in the

calculation of planetary influence improves predictive capability. The hypothesis was tested at decadal, centennial and millennial time scales with positive results.

At the decadal time scale Planet 9 was shown to improve the coherence of the decadal cycle of solar activity with the most basic parameter of barycentric motion, the Sun to planet centre of mass distance, r_{CM} . Further it was shown that the periodicity of r_{CM9} in this decadal time range, 11.9 years, is much closer to the average solar cycle period than the periodicity of r_{CM8} , which varies predominantly at 19.8 years periodicity, Figure 6.

On the centennial time scale the time variation of most interest is the occurrence of grand minimum in sunspot number. Here it was shown that the basic parameter of barycentric motion, r_{CM9} , varied significantly on this time scale with a periodicity of ~ 178 years, Figure 6, and that the long term minima in r_{CM9} coincided frequently with grand minima in sunspot number, Figure 4 and Figure 7. Conversely the Sun – centre mass distance without Planet 9, r_{CM8} , did not vary significantly at this time scale, Figure 5.

At the millennial time scale the spectrum of r_{CM9} was shown to predict cyclic activity at the 88 year Gleissberg period and at the ~ 2400 years Hallstaatt period, whereas, in the spectrum of r_{CM8} these two important cycles were not evident, Figure 8 and Figure 9. An intriguing observation while investigating the millennial time scale of solar activity was that the dominant presence of the ~ 176 year Jose periodicity in planetary motion was, apparently, not reflected in the periodicity of millennium scale solar activity. However, this conundrum was resolved with the discovery, outlined in Section 7, that the Jose periodicity does play a very significant role in millennium scale solar activity but is not directly evident in solar activity spectra because it is phase modulated by a series of longer period components, Figures 13 and 14. Further, it was suggested that all of the long term cyclic variability in sunspot number may devolve from a single periodicity, the Jose periodicity, and its harmonics and sub harmonics.

Thus, on every measure considered, including Planet 9 in planetary barycentric dynamics improved its predictive capability for cyclic solar activity thereby supporting the hypothesis of this paper.

We turn now to the fraught question of whether centennial and millennial scale solar activity is a purely random process internal to the Sun or whether the solar activity is influenced by predictable processes associated with planetary motion. The present work provides new and strong evidence supporting the possibility of predictable processes being in play.

As outlined in the previous paragraphs, planetary theory with Planet 9 included is a more reliable predictor of solar activity on multiple time scales. In addition to this supporting evidence the article uncovers the role of Jose periodicity in the millennium scale periodicity of solar activity as recorded by the reconstructed sunspot number series. If, as suggested in

Section 7 of this article the major part of the spectral content of the reconstructed sunspot number devolves from phase modulation of the Jose component this is powerful support for the concept of planetary motion influencing solar activity because the Jose period is the fundamental parameter of planetary barycentric theory. The spectral content of the reconstructed sunspot number has been studied extensively for more than thirty years, e.g. (Sonett 1984, Vasiliev and Dergachev, 2002, Peristykh and Damon 2003, Knudsen et al 2009, Usoskin et al 2016), with interest in amplitude modulation of the low frequency components and in the Gleissberg component. However, the phase modulation of the Jose component that results in multiple sideband components at the heart of the spectrum has, until now, been overlooked. The presence of the ~176 year Jose periodicity in the spectral content leads naturally to associating the 88 year Gleissberg periodicity as the first harmonic of the Jose period and the conclusion that most of the spectral content in the millennium scale reconstructed sunspot number devolves on the Jose periodicity. That one of the longest and most studied records of solar activity contains evidence of a planetary influence must substantially alter the view about planetary influence on solar activity.

The possibility of understanding the occurrence of grand solar minima has been a major motivation for studying the reconstruction of sunspot number from cosmogenic proxies. The consensus opinion is, for example, *“the occurrence of grand minima/maxima is not a result of long term cyclic variations but is defined by stochastic/chaotic processes”*, Usoskin et al (2007). However, such conclusions appear to contradict some of the observations obtained by the authors. For example, Usoskin et al (2007) provide dates for the times of occurrence of grand solar minima as determined from reconstructed sunspot number. If the dates provided are plotted against the nearest integer multiple of -176.5 years beginning at year 2000 the graph below, Figure 17, results. Also plotted is the Hallstaatt cycle (scaled to fit) as determined from the function derived in section 7.

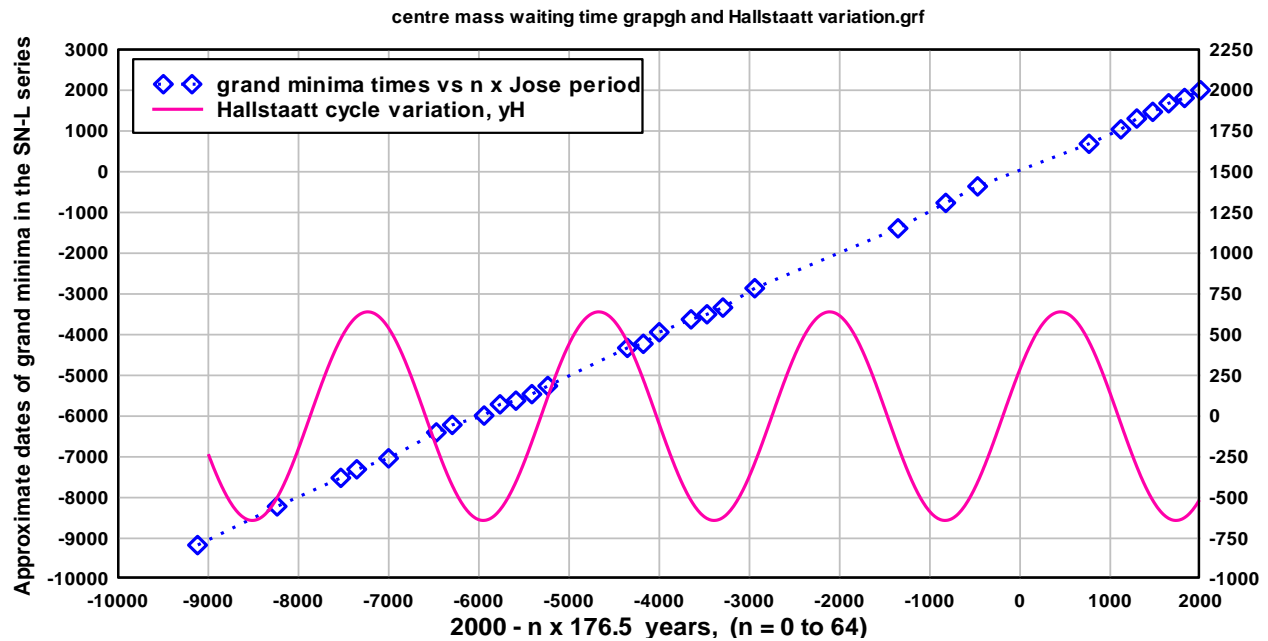


Figure 17. Left hand scale: Dates for the times of occurrence of grand solar minima as determined from reconstructed sunspot number, Usoskin et al (2007), plotted against the dates of $[2000 - n \times (176.5)]$ years, where $n = 0$ to 52. On the right hand scale: A scaled version of the time variation of the approximately 2500 year period Hallstaatt cycle, yH , as determined in Section 7 from the sidebands in the spectrum of reconstructed sunspot number. Note that grand solar minima occur in clusters coincident with the negative phases of the Hallstaatt cycle and, within the clusters, grand solar minima occur, predominantly, at intervals close to 176 years.

It is evident, from Figure 17, that the occurrence of grand minima cluster around the times of Hallstaatt cycle minima, a fact noted by Usoskin et al (2007). It is also evident that the time intervals between grand minima events occurring within these clusters is much less than the waiting time estimated by Usoskin et al (2007), i.e. 330 ± 50 years, and, is much closer to the Jose period ~ 176 years.

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